



Thermodynamic Hadron-Quark Phase Transition of Chiral Nuclear Matter at High Temperature

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Abstract: Based on the extended Nambu-Jona-Lasinio (NJL) model with the scalar-vector eight-point interaction [15], we consider what ultimately happens to exact chiral nuclear matter as it is heated. In the realm of very high temperature the fundamental degrees of freedom of the strong interaction, quarks and gluons, come into play and a transition from nuclear matter consisting of confined baryons and mesons to a state with ‘liberated’ quarks and gluons is expected. In this paper, the hadron-quark phase transition occurs above a limited temperature and after the chiral phase transition in the nuclear matter. There is a so-called quarkyonic-like phase, in which the chiral symmetry is restored but the elementary excitation modes are nucleonic at high density, appears just before deconfinement.

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I. INTRODUCTION

The confinement mechanism is an intrinsic property of quantum chromodynamics (QCD) - the fundamental theory of the strong interaction [1]. As very large temperatures, the interactions which confine quarks and gluons inside hadrons should become sufficiently weak to release them [2]. The phase where quarks and gluons are deconfined is termed the quark-gluon plasma (QGP). Lattice QCD calculations have established the existence of such a phase of strongly interacting matter at temperatures larger than ~ 170 MeV. There have been proposed and discussed various types of scenario concerned with the hadron-quark deconfinement transitions at high-density and high-temperature regions, but it is still unclear, even, whether the phase transition is the cross over or the first order [3]. Only assumption in

this paper is that the phase transition is of the first order as suggested by many model studies [4] and ours [15]. One of the direct consequences of this assumption is the emergence of the hadron-quark (HQ) mixed phase during the phase transition.

The transition between confinement and deconfinement is of the phase transition between hadronic and quark-gluon matters. Theoretical studies of the hadron-quark phase transition and/or the phase diagram on the temperature- chemical potential plane for quark-hadron many-body systems at finite temperature and density are the most recent interests. In these extremely hot and/or dense environment for quark-hadron systems, there may exist various possible phases with rich symmetry breaking pattern [3]. The extremely high density and/or temperature system which

is reproduced experimentally by the relativistic heavy ion collisions (RHIC) has been examined theoretically by the first principle lattice calculations. In the finite density system, however, the lattice QCD simulation is not straightforwardly feasible due to the so-called sign problem, namely, it is difficult to understand directly from QCD at finite density. Thus, the effective model based on QCD can be a useful tool to deal with finite density system. By using the various effective models, the chiral phase transition has been often investigated at finite temperature and density. However, it is still difficult to derive the definite results on the quark-hadron phase transition due to the quark confinement on the hadron side.

For the symmetric nuclear matter, it is important to describe the properties of nuclear saturation and chiral symmetry restoration. The Walecka model [5] has succeeded in describing the saturation property of symmetric nuclear matter as a relativistic system. The underlying microscopic mechanism for saturation is a competition between attractive and repulsive forces among nucleons, with the attraction winning at this particular value of the baryon density. Although this model has given many successful results for nuclei and nuclear matter, this model at first stage has no chiral symmetry which plays an important role in QCD. The NJL model [6] is one of the useful effective models of QCD. The celebrated NJL model [6] gives many important results for hadronic world [7] based on the concepts of the chiral symmetry and the dynamical chiral symmetry breaking. This model has been applied to the investigation of the dense quark matter [8]. Also, by using this model, the stability of nuclear matter, as well as quark matter, was investigated in which the nucleon is constructed from the viewpoint of quark-diquark picture [9], and beyond mean-field theory [10, 11, 12]. On the other hand, it is known that, if the

nucleon field is regarded as a fundamental fermion field, not composite one, the nuclear saturation property cannot be reproduced starting from the original NJL model with chiral symmetry. However, if the scalar-vector and isoscalar-vector eight-point interactions are introduced holding the chiral symmetry in the original NJL model, the nuclear saturation property is well reproduced [13] where the nucleon is treated as a fundamental fermion. Recently, we reconsidered the possibility of using an extended version of the NJL model including in addition a scalar-vector interaction in order to describe chiral nuclear matter at finite temperature and the phase structures of the liquid-gas transition [14] and chiral transition [15]. This ENJL (Extended Nambu–Jona–Lasinio) version reproduces well the observed saturation properties of nuclear matter such as equilibrium density, binding energy, compression modulus, and nucleon effective mass at $\rho_B = \rho_0$. It reveals a first-order phase transition of the liquid-gas type occurring at subsaturated densities; such a transition is present in any realistic model of nuclear matter; The model [15] predicts a restoration of chiral symmetry at high baryon densities, $\rho_B \geq 2.2 \rho_0$ for $T \leq 171$ MeV, and at high temperatures $T > 171$ MeV for $\rho_B < 2.2 \rho_0$.

For the quark-gluon matter, we use the effective models of QCD such as the MIT (Massachusetts Institute of Technology) bag model or the NJL model for quark matter have been actively done instead. We, hereafter, use the MIT bag model for simplicity. The QCD undergoes a phase transition at high temperatures, to the so-called quark-gluon plasma phase. By studying how hadrons melt we may learn more about their structure. So, hadrons have to be melted first, before filling the space with thermal quarks and gluons.

In this paper, the nuclear matter equations of state used in [15] featured a first

order phase transition at high temperatures between hadronic matter, described by phenomenological equations of state, and the quark-gluon plasma (QGP), described by the MIT bag model. We then construct a nuclear matter EoS (Equations of State) similar to that of Ref. [15] in equilibrium with the MIT bag EoS [16] for the QGP phase at high temperatures. In the high-temperature results, it is expected that a quark-hadron phase transition occurs after the chiral symmetry restoration in nuclear matter.

This paper is organized as follows. In the next section, we briefly recapitulate the extended NJL model at finite temperature and baryon chemical potential for nuclear following Ref. [15]. In Sec. III, the quark-hadron phase transition at high temperature is described based on this model. The last section is devoted to a summary and concluding remarks.

II. THE CHIRAL NUCLEAR MATTER

For hadronic matter we use a modification of the original $\sigma - \omega$ model [5], which was presented in Ref. [15]. For the original $\sigma - \omega$ model, the EoS, i.e., the pressure P as a function of the independent thermodynamical variables temperature T and baryochemical potential μ , can be derived from the Lagrangian employing the meanfield (or Hartree, or one-loop) approximation of quantum many-body theory at finite temperature and density.

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\hat{\partial} + \mu\gamma_0)\psi + \frac{G_s}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] \\ & - \frac{G_v}{2}[(\bar{\psi}\gamma^\mu\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\psi)^2] \\ & + \frac{G_{sv}}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2][(\bar{\psi}\gamma^\mu\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\psi)^2], \end{aligned} \quad (1)$$

where $\tau = \sigma/2$ with σ Pauli matrices, μ is the baryon chemical potential, and G_s , G_v and G_{sv} are coupling constants.

At nuclear scale, fermion interactions are in bound states as so-called bosonization,

$$\begin{aligned} \sigma &= \bar{\psi}\psi, \quad \vec{\pi} = \bar{\psi}i\gamma_5\vec{\tau}\psi, \quad \omega_\mu = \bar{\psi}\gamma_\mu\psi, \quad \phi_\mu = \bar{\psi}\gamma_5\gamma_\mu\psi. \\ &\text{yielding} \end{aligned}$$

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\hat{\partial} + \mu\gamma_0)\psi + [G_s + G_{sv}(\omega^2 + \phi^2)]\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\vec{\pi})\psi \\ & - [G_v - G_{sv}(\sigma^2 + \pi^2)]\bar{\psi}(\hat{\omega} + \gamma_5\hat{\phi})\psi \\ & - \frac{G_s}{2}(\sigma^2 + \pi^2) + \frac{G_v}{2}(\omega^2 + \phi^2) \\ & - 3\frac{G_{sv}}{2}(\sigma^2 + \pi^2)(\omega^2 + \phi^2). \end{aligned} \quad (2)$$

In the mean-field approximation, the σ (scalar), π (iso-scalar), ω (vector), and ϕ (iso-vector) fields have the ground state expectation values

$$\langle\sigma\rangle = u, \quad \langle\pi_i\rangle = 0, \quad \langle\omega_\mu\rangle = \rho_B\delta_{0\mu}, \quad \langle\phi_\mu\rangle = 0. \quad (3)$$

Hence,

$$\mathcal{L}_{MFT} = \bar{\psi}(i\hat{\partial} - m^* + \gamma_0\mu^*)\psi - U(\rho_B, u), \quad (4)$$

where

$$m^* = -\bar{G}_s u, \quad \bar{G}_s = G_s + G_{sv}\rho_B^2, \quad (5)$$

$$\mu^* = \mu - [G_v - G_{sv}(u^2 + v^2)]\rho_B, \quad (6)$$

$$U(\rho_B, u) = \frac{1}{2}(G_s u^2 - G_v \rho_B^2 + 3G_{sv} u^2 \rho_B^2). \quad (7)$$

Based on Lagrangian (4) the thermodynamic potential is derived

$$\Omega(\rho_B, u) = U(\rho_B, u) + 2N_f \int \frac{d^3k}{(2\pi)^3} [E_k + T \ln(n_- n_+)], \quad (8)$$

where

$$n_\mp = [e^{E_\mp/T} + 1]^{-1}, \quad E_\mp = E_k \mp \mu^*, \quad E_k = \sqrt{k^2 + m^{*2}},$$

and $N_f = 2$ for nuclear matter and $N_f = 1$ for neutron matter.

The ground state of nuclear matter is determined by the minimum condition

$$\frac{\partial\Omega}{\partial u} = 0$$

or

$$u = 2N_f \int \frac{d^3k}{(2\pi)^3} \frac{m^*}{E_k} (n_- + n_+ - 1), \quad (9)$$

which is called the gap equation.

In terms of the baryon density

$$\rho_B = -\frac{\partial\Omega}{\partial\mu_B} = 2N_f \int \frac{d^3k}{(2\pi)^3} (n_- - n_+), \quad (10)$$

the EoS read

$$P = -\frac{m^{*2}}{2\tilde{G}_s} - \frac{G_v}{2}\rho_B^2 + (\mu - \mu^*)\rho_B - 2N_f \int \frac{d^3k}{(2\pi)^3} [E_k + T \ln(n_- n_+)], \quad (11)$$

$$\mathcal{E} = \frac{m^{*2}}{2\tilde{G}_s} + \frac{G_v}{2}\rho_B^2 + 2N_f \int \frac{d^3k}{(2\pi)^3} E_k (n_- + n_+ - 1). \quad (12)$$

The model is able to reproduce well-observed saturation properties of nuclear matter such as equilibrium density, binding energy, compression modulus, and nucleon effective mass at the saturation density $\rho_B = \rho_0$. Values of parameters and physical quantities are given in Table 1, based on requiring that

$$m_N = -\tilde{G}_s u_{\text{vac}} = 939 \text{ MeV}, \quad (13)$$

with u_{vac} satisfying the gap equation (9) taken at vacuum, $T = 0$, and $\rho_B = 0$, and

$$\mathcal{E}_{\text{bin}} = -m_N + \mathcal{E}/\rho_B \simeq -15.8 \text{ MeV at } \rho_B \simeq 0.17 \text{ fm}^{-3}. \quad (14)$$

The dependence of the binding energy on baryon density shows in Figure 1

TABLE I. Values of parameters and physical quantities.

	$G_s(\text{fm}^2)$	G_v/G_s	G_{sv}/G_s	m_0	m^*/m_N	$K_0(\text{MeV})$
[5]	9.573	1.219	-	-	0.556	540
[14]	8.507	0.933	1.107	41.26	0.684	285.91
[15]	8.897	0.947	1.073	0	0.663	267.23
Expt.	~ 10.145	~ 1.447	-	-	~ 0.6	200 - 300

The model gives two interesting results. First, it reveals a first-order phase transition of the liquid-gas type occurring at subsaturated densities, starting from $T = 0$, $\mu_B \approx 923$ MeV and extending to a crossover critical end point (CEP) at $T \approx 18$ MeV, $\mu_B \approx 922$ MeV. Second, the model predicts an exact restoration of chiral symmetry at high baryon densities, $\rho_B \geq 2.2 \rho_0$ for $0 \leq T \leq 171$ MeV and $\mu_B \geq 980$ MeV, or at high temperatures $T > 171$ MeV for $\mu_B \leq 980$ MeV and $\rho_B < 2.2 \rho_0$.

In the (T, μ_B) plane a second-order chiral phase transition occurs at $T = 0$, $\mu_B \approx 980$ MeV and extends to a tricritical point CP at $T \approx 171$ MeV, $\mu_B \approx 980$ MeV, signaling the onset of a

first-order phase transition for $T \geq 171$ MeV. The phase diagram of the two features is displayed in Figure 2. It displays a clear first-order liquid-gas transition of symmetric nuclear matter at subsaturation and a chiral phase transition of nuclear matter at high baryon density (with the second-order) or at high temperature (with the first-order).

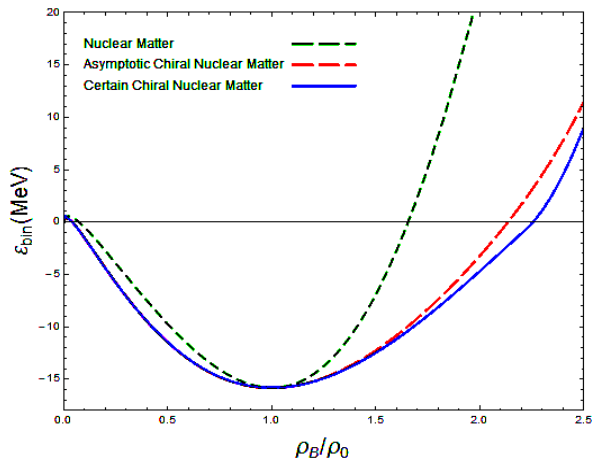


Fig. 1. (Color online) Nuclear binding energy as a function of baryon density. The green short dashed, red long dashed, and blue solid lines are taken from Refs. [5], [14], and [15], respectively

III. THE HADRON-QUARK PHASE TRANSITION AT HIGH TEMPERATURE

In this section we discuss the emergence of the inhomogeneous structure associated with the hadron-quark deconfinement transition. For this purpose we need both EOSs of hadron matter and quark-gluon plasma as realistically as possible. As we mentioned in the last section, no one knows how to exactly calculate the hadron-quark phase transition at high temperature regions. The studies by using the effective models of QCD such as the MIT bag model or the NJL model have been actively done instead, we here use the MIT bag model for simplicity.

A. Hadron phase at chiral limit and high temperature

We now study the chiral phase transitions at high temperature. Form the phase diagram

(Fig. 2) and ρ_B dependence of the chiral condensate (Fig. 3), we realize that the chiral phase transition at high temperature is the first-order and above $T \approx 171$ MeV. For example at $T = 190$ MeV, the shadow region shows that the chiral condensate is a multivalued function and that it is a mixture state of hot nuclear phase and hot chiral phase.

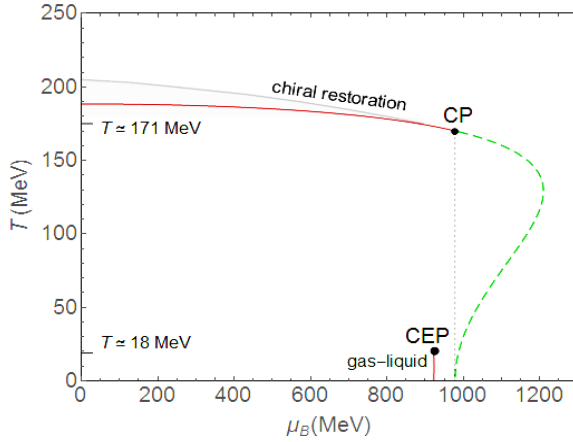


Fig. 2. (Color online) The phase transitions of the chiral nuclear matter in the (T, μ_B) plane. The solid line means a first-order phase transition. CEP ($T \approx 18$ MeV, $\mu_B \approx 922$ MeV) is the critical end point. The dashed line denotes a second-order transition. CP ($T \approx 171$ MeV, $\mu_B \approx 980$ MeV) is the tricritical point, where the line of first-order chiral phase transition meets the line of second-order phase transition. The shadow region is the emergence of hadron-quark mixed phases during the hot chiral phase transition.

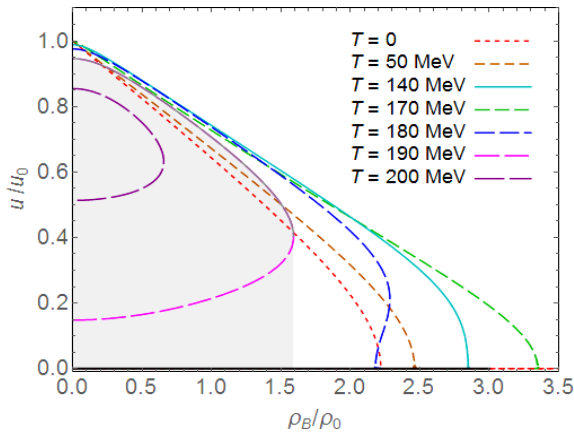


Fig. 3. (Color online) The ρ_B dependence of the chiral condensate at various values of T . For example at $T = 190$ MeV, the shadow region shows that there exists a mixture state of hot nuclear phase and hot chiral phase.

Hence, the integral terms in thermodynamic potential, gap equation, baryon density, energy density and EoS can be expanded about the chiral limit. Thus, Eqs. (9), (10), (11), and (12) lead

$$u \simeq u_{\text{vac}} - \frac{N_f}{\pi^2} \bar{G}_s u [\zeta(2)T^2 - \zeta(0)\mu^{*2}], \quad (15)$$

$$\rho_B \simeq \frac{N_f}{\pi^2} [2\zeta(2)\mu^{*2}T^2 - \frac{2}{3}\zeta(0)\mu^{*3}], \quad (16)$$

$$\begin{aligned} \mathcal{E}_{\text{HD}} &\simeq -\frac{G_s}{2}u^2_{\text{vac}} + \frac{G_v}{2}\rho_B^2 \\ &\quad + \frac{3N_f}{2\pi^2} [7\zeta(4)T^4 + 2\zeta(2)\mu^{*2}T^2 - \frac{1}{3}\zeta(0)\mu^{*4}], \quad (17) \\ P_{\text{HD}} &\simeq -\frac{G_s}{2}u^2_{\text{vac}} + \frac{G_v}{2}\rho_B^2 \\ &\quad + \frac{N_f}{2\pi^2} [7\zeta(4)T^4 + 2\zeta(2)\mu^{*2}T^2 - \frac{1}{3}\zeta(0)\mu^{*4}]. \quad (18) \end{aligned}$$

The Figs. 2 and 3 show that when $T > 171$ MeV the chiral condensate can be dropped to zero even at very small values of the chemical potential and/or baryon density. This is suggested that, when matter is sufficiently heated, hadrons become massless and begin to overlap and quarks and gluons can travel freely over large space-time distances. Within this picture, $T \approx 171$ MeV is the limiting temperature for the deconfinement phase transition between hadrons and quarks and gluons, that we may call the chiral limit.

At high temperature where the chiral symmetry is restored and nucleons become deconfinement, this transition is the so-called quark-hadron transition. Even when the net-baryon number density is small, nuclear matter consists not only of nucleons but also of other, thermally excited hadrons, the lightest hadrons, the pions, are most abundant, the typical momentum scale for scattering events between hadrons is set by the temperature T . If the temperature is on the order of or larger than Λ_{QCD} , scattering between hadrons starts to probe their quark-gluon substructure. Moreover, since the particle density increases with the temperature, the hadronic wave functions will start to overlap for large

temperatures. Consequently, above a certain temperature one expects a description of nuclear matter in terms of quark and gluon degrees of freedom to be more appropriate.

The picture which emerges from these considerations is the following: for very small baryon chemical potentials $\mu_B \sim 0$, the limiting temperature for hadron-quark phase transition from nuclear matter is a gas of hadrons to plasma of quarks and gluons, corresponding to $P \geq 0$, reads

$$T_{\min} = \frac{\sqrt{3}}{\pi} \left(\frac{5}{7} \right)^{1/4} \Lambda \simeq 202.7 \text{ MeV at } \mu_B = 0. \quad (19)$$

B. Quark phase

For the quark phase we employ the standard MIT bag model [16] for massless, non-interacting gluons and u, d quarks. At high temperature EoS of quark-gluon plasma is obtained, i.e.,

$$P_{\text{QCP}} \simeq \frac{8\pi^2 T^4}{45} + N_f \left(\frac{7\pi^2 T^4}{60} + \frac{\mu_q^2 T^2}{2} + \frac{\mu_q^4}{4\pi^2} \right) - B. \quad (20)$$

and other quantities

$$\mathcal{E}_{\text{QCP}} \simeq \frac{8\pi^2 T^4}{15} + N_f \left(\frac{7\pi^2 T^4}{20} + \frac{3\mu_q^2 T^2}{2} + \frac{3\mu_q^4}{4\pi^2} \right) + B, \quad (21)$$

$$\rho_{\text{QCP}} \simeq \frac{N_f}{3} \left(\mu_q T^2 + \frac{\mu_q^3}{\pi^2} \right). \quad (22)$$

Here, a baryon consists of three quarks, $\rho_B = \rho_q/3$ and $\mu_B = 3\mu_q$. To the factor $37 = 16 + 21$, 16 gluonic (8×2), 12 quark ($3 \times 2 \times 2$) and 12 antiquark degrees of freedom contribute, with assuming that only up and down flavors contribute significantly to the quark pressure. The properties of the physical vacuum are taken into account by the bag parameter B , which is a measure for the energy density of the vacuum.

It has been found [17] that within the MIT bag model (without color superconductivity) with a density-independent bag constant B , the maximum mass of a neutron star cannot exceed a value of about 1.6

solar masses. Indeed, the maximum mass increases as the value of B decreases, but too small values of B are incompatible with a hadron-quark transition density $\rho_B > 2.3 \rho_0$ in nearly symmetric nuclear matter, as demanded by heavy-ion collision phenomenology.

In order to overcome these restrictions of the model, one can introduce a density-dependent bag parameter $B(\rho_B)$, and this approach was followed in Ref. [18]. This allows one to lower the value of B at large density (and high temperature), providing a stiffer QGP EoS and increasing the value of the maximum mass, while at the same time still fulfilling the condition of no phase transition below $\rho_B \approx 2 \rho_0$ in symmetric matter. In the following we present results based on the MIT model using a gaussian parametrization for the density dependence,

$$B(\rho_q) = B_\infty + (B_0 - B_\infty) e^{-\beta^2 \left(\frac{\rho_q}{\rho_0} \right)^2}, \quad (23)$$

with $\beta = 0.17$.

The limiting temperature for hadron-quark phase transition, corresponding to $P \geq 0$, reads

$$T_{\min} = \sqrt{\frac{3}{\pi}} \left(\frac{10}{37} \right)^{1/4} B_0^{1/4} \text{ at } \mu_B = 0. \quad (24)$$

Comparing this equation to (19), we get

$$B_0^{1/4} = \left(\frac{37}{14} \right)^{1/4} \frac{\Lambda}{\sqrt{\pi}} \simeq 287.7 \text{ MeV}. \quad (25)$$

The value of B_∞ is fixed at the tricritical point ($T \approx 171 \text{ MeV}$, $\mu_B \approx 980 \text{ MeV}$). It gives

$$B_\infty^{1/4} = \left(\frac{7}{20} \right)^{1/4} \frac{\Lambda}{\sqrt{\pi}} \simeq 173.6 \text{ MeV}. \quad (26)$$

The range of the bag parameters B is found from $B^{1/4} = 125 \text{ MeV}$ to about 300 MeV which is consistent with the results from a bag model analysis of hadron spectroscopy [19].

The hadron-quark transition at very high

temperature provides the following picture: when matter is heated, nuclei eventually dissolve into protons and neutrons (nucleons). At the same time light hadrons (preferentially pions) are created thermally, which increasingly fill the space between the nucleons. Because of their finite spatial extent the pions and other thermally produced hadrons begin to overlap with each other and with the bags of the original nucleons such that a network of zones with quarks, antiquarks and gluons is formed. At a certain critical temperature T_C these zones fill the entire volume in a percolation transition. This new state of matter is the quark-gluon plasma (QGP). The vacuum becomes trivial and the elementary constituents are weakly interacting. There is, however, a fundamental difference to ordinary electromagnetic plasmas in which the transition is caused by ionization and therefore gradual. Because of confinement there can be no liberation of quarks and radiation of gluons below the critical temperature. Thus a relatively sharp transition is expected.

In the MIT-Bag model thermodynamic quantities such as energy density and pressure can be calculated as a function of temperature and quark chemical potential (or baryon chemical potential) and the phase transition is inferred via the Gibbs construction of the phase boundary. By construction, the hadron-quark transition in the MIT bag model is of first order, implying that the phase boundary is obtained by the requirement that, at constant chemical potential, the pressure of the QGP is equal to that in the hadronic phase.

C. Phase equilibrium

The QGP EoS (20) is matched to the hadronic EoS (18) via Gibbs conditions for (mechanical, thermal, and chemical) phase equilibrium [20],

$$P_{\text{HD}} = P_{\text{QGP}}, \quad T_{\text{HD}} = T_{\text{QGP}}, \quad \mu_{\text{HD}} = \mu_{\text{QGP}}, \quad (27)$$

Which leads to a phase boundary curve $T^*(\mu^*)$ in the T - μ plane defined by the implicit equation $P_{\text{HD}}(T^*, \mu^*) = P_{\text{QGP}}(T^*, \mu^*)$, see Fig. 4. The phase transition constructed via (27) is of first order for $T > 171$ MeV, leading to a mixed phase of QGP and hadron matter.

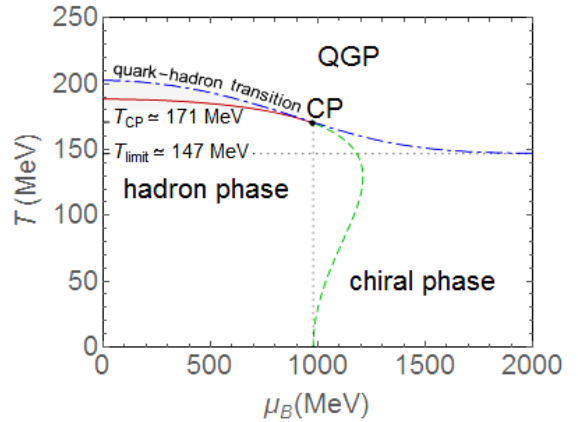


Fig. 4. (Color online) The hadron-quark phase transitions (blue dot-dashed line) of the hot chiral nuclear matter to quark-gluon plasma in the (T, μ_B) plane. The shadow region is the emergence of hadron-quark mixed phases during the hot chiral phase transition.

Here, it should be noted that the quark-hadron phase transition happens above a limited temperature, so there is a region outside chiral symmetry restoration and below the limited temperature, i.e. occurs at densities greater than that for the chiral transition. This suggests that a phase that is chiral symmetric but confined with nucleonic (hadronic) elementary excitation could exist just before the phase transition from the nuclear phase to the quark one. Recently, McLerran and Pisarski have proposed a new state of matter, the so-called quarkyonic matter [21], which is a phase characterized by chiral symmetry restoration and confinement based on large N_c arguments. The name ‘quarkyonic’ expresses the fact that the matter is composed of confined baryons yet behaves like chirally symmetric quarks at high densities. There may be non-perturbative effects associated with confinement and chiral symmetry restoration near the fermi surface, since there the interactions are sensitive to long distance

effects, but the bulk properties should look like almost free quarks.

As shown in Fig. 4, there are certainly two phase boundary, one between the hadronic and quarkyonic phases, and other between the quarkyonic and deconfined phases with a tricritical point $T_{CP} \approx 171$ MeV. Thus, this chiral symmetric nuclear phase predicted by our model may possibly correspond to the quarkyonic phase.

IV. CONCLUSION

The hadron-quark phase transition at very high temperature has been investigated following Ref. [15] in the extended NJL model with scalar-vector eight-point interaction. In this model, as a first attempt to investigate the hadron- quark phase transition, the hadron side was regarded as chiral nuclear matter and the quark side as a quark-gluon plasma with no quark-pair correlation. Both phases were matched via Gibbs' phase equilibrium conditions for a first order phase transition.

There is an interesting phase from the phase diagram in Fig. 4 lying below the quark-hadron phase transition and occurring after chiral symmetry restoration in the nuclear matter. This might appear as an exotic phase, i.e., the nuclear phase, not the quark phase, while the chiral symmetry is restored in terms of the nuclear matter. This phase may possibly correspond to the quarkyonic phase, which is introduced as a chiral symmetric confined matter [21].

In this paper, we have ignored the color superconducting phase which may exist in finite density systems and relate to quarkyonic phase. So, the next challenging task may be to investigate the phases of nuclear matter, including nuclear super-fluidity and quark-gluon plasma, and also including the color superconducting state, i.e., nucleon pairing on the nuclear phase side and quark pairing on the

quark phase side. Further, it is widely believed that neutron star matter undergoes a phase transition to quark-gluon plasma at high temperature and/or density. Thus, it is also interesting to investigate the phase transition between neutron star matter and quark matter. This leads to the understanding and development of the physics of neutron stars.

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