Nucl. Sci. and Tech., Vol.9, No. 4 (2019), pp. 34-40

Nuclear Science and Technology

Journal homepage: https://jnst.vn/index.php/nst

Evaluating uncertainty of some radiation measurand using Monte Carlo method

Bui Duc Ky*, Nguyen Ngoc Quynh, Duong Duc Thang, Le Ngoc Thiem, Ho Quang Tuan,
Tran Thanh Ha, Bui Thi Anh Duong, Nguyen Huu Quyet, Duong Van Trieu

Institute for Nuclear Science and Technology, Hanoi, Vietnam
Email: Duckyb2@gmail.com

Abstract: Evaluating measurement uncertainty of a physical quantity is a mandatory requirement for laboratories within the recognition ISO/IEC 17025 certification to access reliability of measured results. In this work, the uncertainty of ionizing radiation measurements such as air-kerma, personal dose equivalent $Hp(d)$ was evaluated based on GUM method and Monte Carlo method. An uncertainty propagation software has been developed for evaluation of the measurement uncertainty more convenient.

Key words: *Uncertainty measurement, Monte Carlo method.*

I. INTRODUCTION

The measurement uncertainty is a characteristic for the dispersion of measurable values of a quantity to be measured [1, 2, 3]. Because without the measurement uncertainty, the results of the measurements cannot be compared to each other, nor can be compared to conventional true values.

In the field of measurement of ionizing radiation ISO/IEC and IAEA has provided guidance on measurement uncertainty for different measurement quantities. These documents are primarily based on the evaluation methods provided by the International Commission on Measurement Guidelines (JCGM). The uncertainty propagation method described in the JCGM 100:2008 “Guide to the expression of uncertainty in measurement” is often referred to as the GUM method. The Monte Carlo method was described in its supplement 1 “Guide to the expression of uncertainty in measurement – Propagation of distributions using a Monte Carlo method”.

In this work, uncertainty of air-kerma and personal dose equivalent $Hp(d)$ quantities were evaluated by both methods. These two quantities are the fundamental quantities in radiation protection field. All experimental data published in this work were measured at the Secondary Standard Dosimetry Laboratory belongs to Institute for Nuclear Science and Technology.

II. METHODS

The relationship between a single real output quantity y and a number of real input quantities x_i has the following equation (1).

$$y = f(x_1, \dots, x_N) \tag{1}$$

A. GUM method

Uncertainty of the input quantities $u(x_i)$ are divided into two categories, based on how its values were evaluated. If it was evaluated

https://doi.org/10.53747/jnst.v9i4.135
Received 07 November 2019, accepted 26 December 2019

based on statistical means, they are called type A. Otherwise, they are nominated type B. However, it is worth mentioning that this classification does not affect the uncertainty propagation law.

The uncertainty of output quantity Y is calculated as [2, 3, 4]:

$$u^2(y) = \sum_{i=1}^{N} c_i^2 \,.\, u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} c_i \,.\, c_j \,.\, u(x_i, x_j) \quad (2)$$

Where, $c_i = \partial/\partial x_i$ and $c_j = \partial/\partial x_j$ are sensitivity coefficients, $u(x_i, x_j)$ is the estimated covariance associated with x_i and x_j.

The GUM uncertainty framework requires [4]:

a) The non-linearity of the measurement function to be insignificant.

b) The central limit theorem to apply, implying the representativeness of the Probability density function (PDF) for the output quantity by a Gaussian distribution or a t-distribution.

c) The adequacy of the Welch-Satterthwaite formula for calculating the effective degrees of freedom.

In practice, the GUM method is frequently used in violation of the requirements listed above or without knowing whether these requirements hold (with an unquantified degree of approximation). Furthermore, the equation 2 is only the first order Taylor series approximation. This makes calculated uncertainty in many cases inaccurate.

B. Monte Carlo method

The Monte Carlo method simulates input quantities x_i based on initial probability distribution. The distribution of the input quantities will affect the output quantity according to the model in the equation 1. As result, the distribution function of the output quantity was obtained. Therefore, not only the standard deviation but other characteristics of output quantity can be determined (*i.e. skewness, coverage interval*). The process of uncertainty evaluation using Monte Carlo method was presented in fig.1 [2, 4].

The advantages of the Monte Carlo method are that it doesn't make any assumption about linearity of measurement function nor PDF of the output quantity. Therefore, the Monte Carlo method is valid for wider range of problem compare to GUM method. Its result can be used to validate the result of GUM method.

The disadvantage of the Monte Carlo method is that it is impossible using hand calculation. This method must be implemented in a computer software.

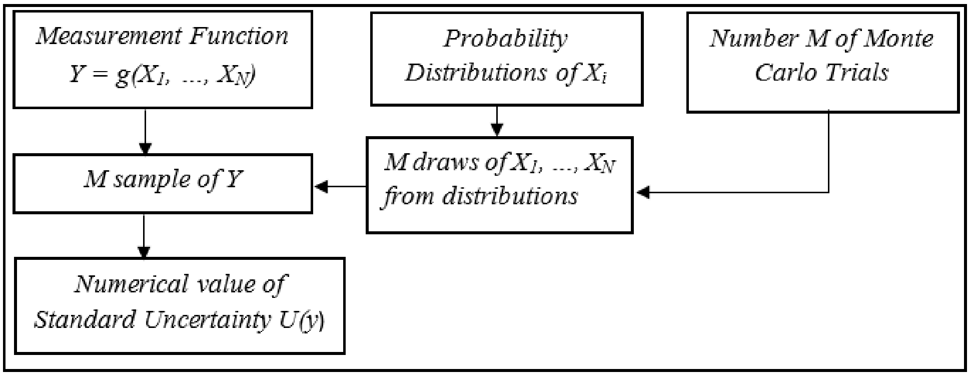
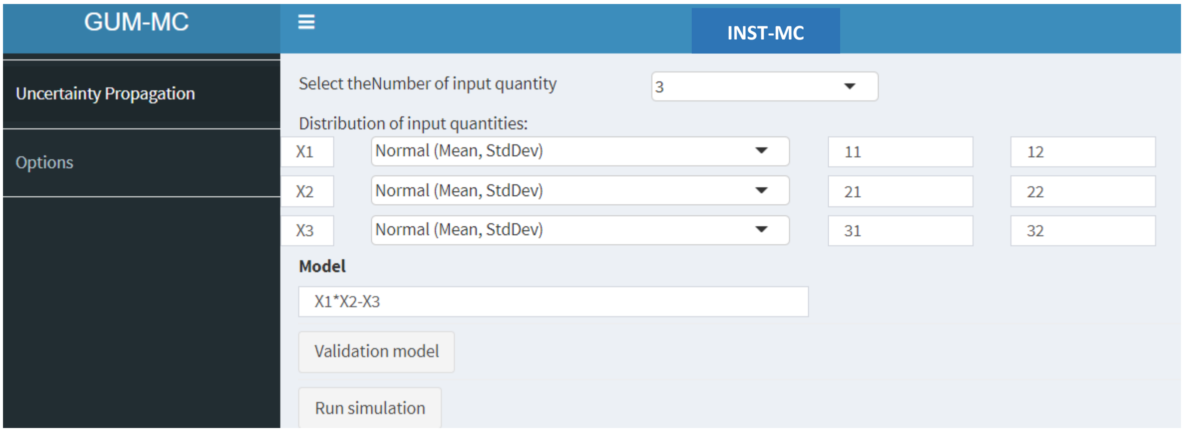


Fig. 1. Uncertainty measurement using a Monte Carlo method for a univariate, real measurement function [4]

C. INST-MC software

A software program, namely INST-MC was developed to facilitate the uncertainty evaluation. The interface of the software is shown in fig.2. Both methods of uncertainty evaluation discussed above were implemented. In the first version of INST-MC, most common distributions in radiation measurement are included: Gaussian distribution, t-distribution, Poisson distribution, uniform distribution, triangular distribution, etc. The program was validated by comparing with the NIST uncertainty machine, an uncertainty software has been developed by National Institute of Standard and Technology/ USA.



Select theNumber of input quantity: 3

Distribution of input quantities:

X1	Normal (Mean, StdDev)	11	12
X2	Normal (Mean, StdDev)	21	22
X3	Normal (Mean, StdDev)	31	32

Model: X1*X2-X3

Fig. 2. Interface of INST-MC uncertainty software

III. RADIATION MEASURAND

A. Air-kerma (K_{air}) of ^{137}Cs source

The air-kerma is obtained from Eq.3

$$K_{air} = N_K * M_{corr} * K_Q \qquad (3)$$

Where: N_K is calibration factor of ionization chamber, K_Q is correction factor of the difference between the reference beam quality,Q_0, and the actual quality, Q, during the measurement and M_{corr} is corrected reading of ionization chamber:

$$M_{corr} = M_{raw} k_T k_P k_{stab} k_{dis} k_{other} \qquad (4)$$

M_{raw} is reading of ionization chamber, $k_T = \frac{T}{T_0}$ corrects for the deviation of the actual air temperature T from the reference temperature $T_0 = 293.15\,\mathrm{K}$, $k_P = \frac{P_0}{P}$ corrects for the deviation of the actual air pressure P from the reference temperature $P_0 = 1013\,\mathrm{mbar}$, k_{stab} corrects for the unstable of ionization chamber, k_{dis} corrects for the possible deviation of the actual distance of the reference source to the measuring instrument from the nominal calibration distance.

Using the equation 2, uncertainty of air-kerma *U(K)* and corrected reading $U(M_{corr})$ is given by:

$$\frac{U(K)}{K} = \sqrt{(\frac{U(N_k)}{N_k})^2 + (\frac{U(M_{corr})}{M_{corr}})^2 + (\frac{U(K_Q)}{K_Q})^2} \qquad (5)$$

$$\frac{U(M_{corr})}{M_{corr}} = \sqrt{\left(\frac{U(M_{raw})}{M_{raw}}\right)^2 + \left(\frac{U(K_T)}{K_T}\right)^2 + \left(\frac{U(K_P)}{K_P}\right)^2 + \left(\frac{U(K_{Stab})}{K_{Stab}}\right)^2 + (\frac{U(K_{dis})}{K_{dis}})^2 + \left(\frac{U(K_{other})}{K_{other}}\right)^2} \qquad (6)$$

B. Personal dose equivalent $Hp(d)$ using TLD dosimeter

Personal dose equivalent Hp(d) is obtained from equation (7):

$$H_p(d) = \frac{(M - M_B).ECC}{RCF} f_E f_{lin} f_d f_{ang} f_{other} \quad (7)$$

Where: M is reading of exposed dosimeter, M_B is reading of background dosimeter.

ECC is elements correction coefficients.

$$ECC = \frac{\bar{R}}{R_i} \quad (8)$$

$\bar{R}$ is average reading of n dosimeters and R_i is reading of ith dosimeter ($i = \overline{1-n}$).

RCF is reader calibration factor:

$$RCF = \frac{C - C_B}{H_C} \quad (9)$$

C is reading of calibration set, C_B is reading of background calibration set, H_C is conventional true value (exposed dose), f_E corrects for energy dependence of dosimeter, f_{lin} corrects for non-linearity of dosimeter, f_d corrects for the loss signal before reading dosimeter, f_{ang} corrects for the inhomogeneity response of dosimeter, f_{other} corrects for others affect.

The uncertainty of personal dose equivalent $Hp(d)$ is obtained from equation 10.

$$\frac{U(H_P(d))}{H_P(d)} = \sqrt{\begin{array}{c} \left(\frac{U(M-M_0}{M-M_0}\right)^2 + \left(\frac{U(ECC)}{ECC}\right)^2 + \\ \left(\frac{U(RCF)}{RCF}\right)^2 + \left(\frac{U(f_E)}{f_E}\right)^2 + \\ \left(\frac{U(f_{lin})}{f_{lin}}\right)^2 + \left(\frac{U(f_{ang})}{f_{ang}}\right)^2 + \left(\frac{U(f_{other})}{f_{other}}\right)^2 \end{array}} \quad (10)$$

IV. RESULTS AND DICUSSION

A. Uncertainty of air-kerma with the gamma rays of ^{137}Cs

The measurement uncertainty of air-kerma U_{kair} using approximation method estimated around 1.26%. The uncertainty corresponds to a coverage factor k = 1 and a level of confidence factor of approximately p = 68%. The details of uncertainty of components showed in Table I.

Table I. Uncertainty budget of air-kerma

Source of uncertainty	Relative standard deviation (%)	Type of uncertainty	Degree of freedom
Calibration factor of ionization chamber	0.41	B	-
Reading of ionization chamber	0.10	A	9
Air pressure	0.11	B	-
Air temperature	0.10	B	-
Distance	0.13	B	-
Stability of ionization chamber	0.60	A, B	-
Others	1.00	A, B	-
U_c	***1.26***		

Calculation model of air-kerma using INST-MC software is given by equation (11):

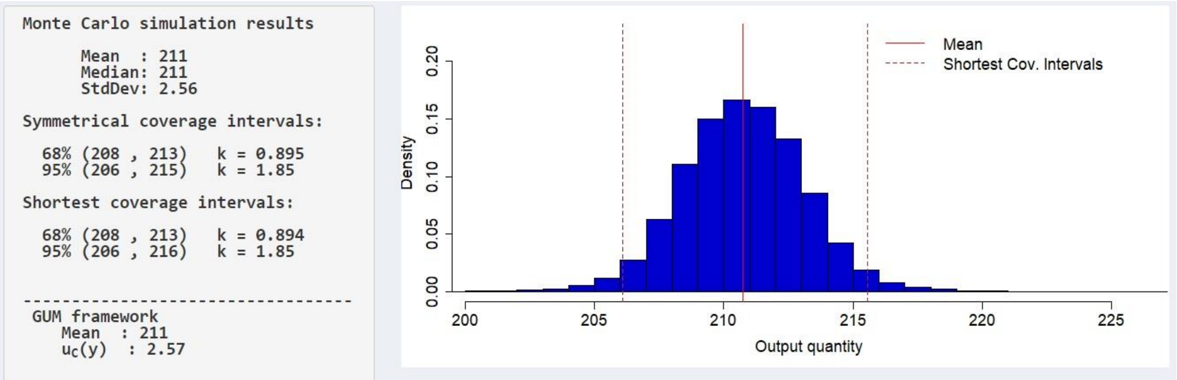
$$K_{air} = N_K . M_{raw} . k_T . k_P . k_{stab} . k_{dis} \qquad (11)$$

Based on data in table III, Monte Carlo method estimated measurement uncertainty of air-kerma approximately 1.21% correspond a coverage factor k = 1 and a level of confidence factor of approximately p = 68%.

Table II. Distribution of input quantities of air- kerma

Input quantities, X_i	**Average value of X_i**	**Standard deviation**	**Distribution**	**Degree of freedom**
Reading of ionization chamber, M_{raw}	4.175	0.035	Student	3
Calibration factor of ionization chamber, N_K	50.23	0.27	Student	3
K_P	0.997	0.007	Rectangular	-
K_T	1.008	0.0006	Rectangular	-
K_{Stab}	1.001	-	Constant	-
K_{dis}	0.999	-	Rectangular	-

Figure 3 show the detail of the uncertainty result of air-kerma using Monte Carlo method. The uncertainty evaluation results of two method are very close. That means GUM method is valid, and uncertainty of air-kerma can be calculated by either GUM method or Monte Carlo method.



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Monte Carlo simulation results

      Mean  : 211
      Median: 211
      StdDev: 2.56

Symmetrical coverage intervals:

  68% (208 , 213)   k = 0.895
  95% (206 , 215)   k = 1.85

Shortest coverage intervals:

  68% (208 , 213)   k = 0.894
  95% (206 , 216)   k = 1.85


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 GUM framework
    Mean  : 211
    uc(y)  : 2.57
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Fig. 3. Measurement uncertainty result of air kerma using INST-MC software

B. Uncertainty of dose equivalent $Hp(d)$ using TLD dosimeters

The measurement uncertainty of dose equivalent $U_{Hp(d)}$ using approximation method estimated around 18.1%. The uncertainty corresponds to a coverage factor k = 1 and a level of confidence factor of approximately p = 68%. The details of uncertainty of components are showed in Table III.

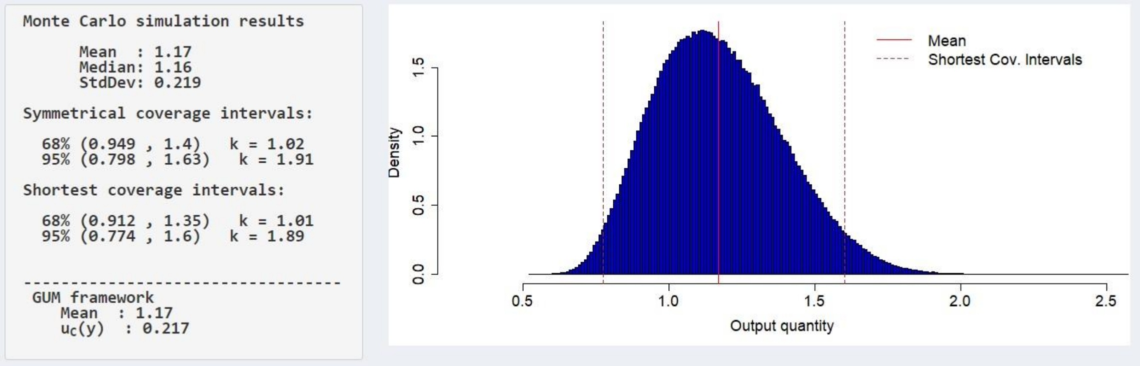
Table III. Uncertainty budget of dose equivalent $Hp(d)$

Source of uncertainty	Relative standard deviation (%)	Type of uncertainty	Degree of freedom
Conventional true value (exposed dose), *Hc*	2.36	B	-
Reading of dosimeters, *M*	2.7	A	4
Elements Correction Coefficients, *ECC*	2.1	A	99
Reader Calibration Factor, *RCF*	3.5	A, B	9
f_E	12.6	B	-
f_{lin}	6.5	B	-
f_{ang}	9.1	B	-
f_d	1.9	B	-
others	3.0	-	-
$\boldsymbol{U_c}$		***18.1***	

Calculation model of dose equivalent $Hp(d)$ is given by equation (7). Based on distribution of input quantities of personal dose equivalent $Hp(10)$ in table IV, INST-MC estimated measurement uncertainty of dose equivalent approximately 18.7% correspond a coverage factor k = 1 and a level of confidence factor of approximately p = 68%. Fig.4 show the detail of uncertainty of personal dose equivalent Hp(d) using Monte Carlo method.

Table IV. Distribution of input quantities of personal dose equivalent $Hp(10)$

Input quantities, X_i	Value of X_i	Standard deviation	Distribution	Degree of freedom
Reading of dosimeter, $M - M_B$	4817	90.48	Student	5
Elements Correction Coefficients, ECC	0.88	0.07	Student	99
Reading of calibration set, $C - C_B$	15443	545	Student	5
Conventional true value, H_c	6.8	0.082	Student	7
f_E	0.615	0.08	Rectangular	-
f_{lin}	1.02	0.034	Rectangular	-
f_{ang}	1.02	0.091	Rectangular	-
f_d	0.98	0.019	Rectangular	-



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Monte Carlo simulation results

      Mean  : 1.17
      Median: 1.16
      StdDev: 0.219

Symmetrical coverage intervals:

  68% (0.949 , 1.4)   k = 1.02
  95% (0.798 , 1.63)   k = 1.91

Shortest coverage intervals:

  68% (0.912 , 1.35)   k = 1.01
  95% (0.774 , 1.6)   k = 1.89


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 GUM framework
    Mean  : 1.17
    uc(y)  : 0.217
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Fig. 4. Measurement uncertainty result of $Hp(d)$ using INST-MC software

IV. CONCLUSIONS

The measurement uncertainty of air-kerma and the personal dose equivalent $H_p(d)$ were evaluated by the GUM method and Monte Carlo method, which were implemented in the INST-MC software program. The results showed that deviations of air-kerma and personal dose equivalent $Hp\,(d)$ calculated by two methods are 3.9% and 3.3%, respectively. Compared with the approximation method, INST-MC is more convenient to calculate and it also shows the probability distribution of the obtained results.

In further research, uncertainty evaluation of other quantities in SSDL will be estimated by Monte Carlo method.

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