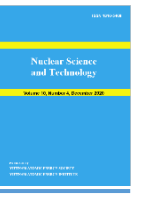
Nucl. Sci. and Tech., Vol.10, No. 2 (2020), pp. 52-59

Nuclear Science and Technology

Journal homepage: https://jnst.vn/index.php/nst



Coupled reaction channels study of the ^{16}O(d,^{6}Li) reaction

Do Cong Cuong, Nguyen Hoang Phuc, Nguyen Tri Toan Phuc

Center for Nuclear Physics, Institute for Nuclear Science and Technology.
179 *Hoang Quoc Viet street, Cau Giay, Hanoii*
Email: dccuong@mail.vinatom.gov.vn; cuong1981us3@gmail.com

Abstract: The α transfer ^{16}O(d,^{6}Li)^{12}C reaction has been studied within the coupled reaction channels (CRC) approach, inluding both the direct and indirect α transfer processes. The obtained results show an important contribution of the indirect α transfer via the 2^+ and 4^+ states of ^{12}C. The CRC results show that the best-fit α spectroscopic factors of ^{16}O becomes smaller when the indirect transfer processes are taken into account. The α spectroscopic factors deduced from the present CRC analysis of the ^{16}O(d,^{6}Li)^{12}C reaction data measured at E_d=54.25 and 80 MeV are quite close to each other.

Keywords: *coupled reaction channels calculations, transfer reactions, α-cluster.*

I. INTRODUCTION

The established α cluster structure of the excited states of ^{12}C at the energies near the α decay threshold are of interest for both the nuclear physics and astrophysics. For example, the isoscalar 0^+_2 excitation of ^{12}C at 7.65 MeV, known as the Hoyle state, plays a vital role in the stellar carbon synthesis. In general, the α cluster models, which describe the nuclear wave functions in terms of the α particles moving in the inter-cluster potential, not only reporduce the main features of these excited states but also show a significant fraction of the α cluster component in the ground state [1-6]. Although ^{16}O nucleus in the ground state (g.s.) is well known to be of the shell-model structure, the α cluster model calculations have predicted the α spectroscopic factor S_α of about $\sim$ 0.3 [1-6] for ^{16}O$_{g.s.}$. Such values of S_α were also confirmed in the shell model calculations [7-10], where the overlap of the α cluster configuration with the total g.s. wave function is calculated exactly. Several measurements have been performed to determine the α spectroscopic factor of ^{16}O$_{g.s.}$ [11-20], but the deduced S_α values are ranging widely from about 0.3 to 1.0, depending on the reaction mechanism and analysis method [11-14, 16-19]. Thus, the α spectroscopic factors of ^{16}O still remain the research topic of different nuclear structure and reaction studies.

Among various experiments, the α transfer reactions like (d,^{6}Li), (t,^{7}Li), (^{3}He,^{7}Be), and (α,^{8}Be) [11-15,20] were proven to be very helpful for the determination of the α spectroscopic factors. The most important inputs for the analysis of a transfer reaction in either the distorted wave Born approximation (DWBA) or the CRC formalism are the α spectroscopic factor S_α and the optical potentials (OP) for both the entrance and exit channels of the reaction. We note that a widely adopted prescription for the DWBA or CRC calculations of a transfer reaction is to use the

https://doi.org/10.53747/jnst.v10i2.37
Received 21 March 2020, accepted 25 June 2020

complex OP of a system of the two colliding nuclei having masses similar to those in the exit channel of the α transfer reaction, at about the same center-of-mass (c.m.) energy. The uncertainty of the deduced S_α values remains, however, significant, and the analysis of the same α transfer reaction happened to deliver different α spectroscopic factors because of the different OP used for the exit channel. Therefore, it is highly desirable to have the OP for both the entrance and exit channels determined from the optical model (OM) analysis of the elastic scattering of the same systems at the nearby energies, to accurately determine the corresponding α spectroscopic factors. In the present work, such a procedure is carried out to determine the α spectroscopic factors of ^{16}O from the CRC analysis of the ^{16}O(d,^{6}Li) reaction.

II. CRC FORMALISM

We give here a brief description of the coupled reaction channels method used in our calculation using the code Fresco written by Thompson [21]. In general, the cross section of the α transfer reaction is given by the solution of the following coupled channel (CC) equations, where the relative wave function of the β channel is determined in the post form as [21,22].

$$\left(E_\beta - T_\beta - U_\beta\right)\chi_\beta = \sum_{\beta',x} \langle\beta|V|\beta'\rangle\chi_{\beta'} + \sum_{\beta'\neq\beta,\,x'}\left[\langle\beta|W_{\beta'}|\beta'\rangle + \langle\beta|\beta'\rangle\left(T_{\beta'} + U_{\beta'} - E_{\beta'}\right)\right]\chi_{\beta'}, \quad (1)$$

With x, x' being the incoming and outgoing partitions, respectively. U_β and $U_{\beta'}$ are the diagonal optical potentials, χ_β and $\chi_{\beta'}$ are the relative-motion wave functions of the corresponding channels. For the α transfer ^{16}O(d,^{6}Li)^{12}C reaction, the transfer interaction is determined in the post form as:

$$W_\beta = V_{12C-\alpha} + (U_{12C-d} - U_{6Li-12C}), \quad (2)$$

Where $V_{12C-\alpha}$ is the potential binding the α cluster to the ^{12}C core in ^{16}O. U_{12C-d} and $U_{6Li-12C}$ are the core-core OP and OP of the exit partition. In our CRC calculation, the nonorthogonality correction and complex remnant term are properly taken into account.

For the α cluster structure of the ^{16}O nucleus, the 1 s state is assumed for the internal state of the constituent α cluster. The number of node N of the relative wave function Φ_{NL} of the α + ^{12}C cluster configuration is given by the Wildermuth condition:

$$2(N-1) + L = \sum_{i=1}^{4} 2(n_i - 1) + l_i, \quad (3)$$

Where l_i and n_i are the orbital angular momentum and number of node, respectively, of each nucleon in the α cluster. The wave function Φ_{NL} is generated by solving a two-body Schrödinger equation with the $V_{12C-\alpha}$ potential in Woods-Saxon form. The depth of this potential is adjusted to reproduce the experimental α separation energy of ^{16}O

$$E_\alpha(I_\alpha^\pi) = E_\alpha(\text{g.s.}) - E(^{16}O^*) + E(^{12}C^*), \quad (4)$$

With $E_\alpha(\text{g.s.})$ is the α separation energy of ^{16}O in the ground state, $E(^{12}C^*)$ and $E(^{16}O^*)$ are the excitation energies of ^{12}C and ^{16}O nuclei, respectively.

One essential input of the CRC calculation for transfer reaction is the spectroscopic factor. The α spectroscopic factor defined as $S_\alpha = |A_{NL}|^2$, is used to construction the overlap function

$$\left\langle ^{12}C\middle|^{16}O\right\rangle = A_{NL}\Phi_{NL}(\boldsymbol{r}_{\alpha+12C}). \quad (5)$$

In the present analysis, the α spectroscopic factor is given by the cluster-nucleon configuration interaction model in *psd* model space [10]. It is important to note this α spectroscopic factor follows the Fliessbach definition, which takes into account microscopic antisymmetrization and orthonomalization effect for the two-body cluster wave function.

III. RESULTS AND DISCUSSION

The α spectroscopic factors of light nuclei are of high interest for both theoretical and experimental studies [1,2,7,11-13]. However, those values of the α spectroscopic factors in these nuclei are uncertain and seem to depend on the direct reaction mechanism as well as on the theoretical models used in the analysis. For example, the α spectroscopic factors deduced from the transfer reactions are smaller than those obtained from the ($p,p\alpha$) knock-out reactions [13,15,16,23,24]. It is, therefore, of interest for the present research to determine the α spectroscopic factor of the ^{16}O nucleus from the CRC analysis of the α transfer reaction ^{16}O(d,^{6}Li)^{12}C reaction using the optical potentials that give good OM description of the elastic d+^{16}O and ^{6}Li+^{12}C scattering at the considered energies [15, 20].

Table I. The WS parameters of the complex OP used in the present CRC analysis for the elastic of d+^{12}C and d+^{16}O scattering at E_d = 54.3 MeV and elastic ^{6}Li+^{12}C scattering at E_{6Li} = 63 MeV

System	V_0 (MeV)	r_V (fm)	a_v (fm)	W_0 (MeV)	W_S (MeV)	r_W (fm)	a_w (fm)
d+^{12}C[a]	71.8	1.25	0.700		11.0	1.25	0.700
d+^{16}O[a]	68.2	1.25	0.693		10.2	1.25	0.790
^{6}Li+^{12}C	160.5	1.15	0.750	11.0		2.27	0.650

[a] The WS parameters taken from OM analysis of the elastic d+^{12}C scattering at 52 MeV [25]

In CRC calculations, quite important are the inputs of the OP, which generate the scatteing waves in both entrance and exit channels, and are used in the remnant term (2). For the ^{16}O(d,^{6}Li)^{12}C reaction, the OP's of deuteron on the ^{12}C and ^{16}O targets used for the core-core interaction and the entrance channel are assumed to be the same as the OP's of the d+^{12}C and d+^{16}O systems at E_{lab} = 52 MeV, which have been adjusted to reproduce the elastic scattering data [25]. The optical potentials used in the present CRC calculations are determined as Woods-Saxon form:

$$U(r) = V_0 f(r, R_V, a_V) + iW_0 f(r, R_W, a_W) + iW_S \frac{d}{dr}[f(r, R_W, a_W)], \quad (6)$$

And Coulomb potentials of a uniform charged sphere,

$$V_C(r) = \begin{cases} \frac{Z_P Z_T e^2}{2R_C}\left(3 - \frac{r^2}{R_C^2}\right), & r \le R_C, \\ \frac{Z_P Z_T e^2}{r}, & r > R_C. \end{cases} \quad (7)$$

Here:

$$f(r, R_i, a_i) = \left[1 + exp\left(\frac{r - R_i}{a_i}\right)\right]^{-1}, \quad (8)$$

$$R_i = r_i\left(A_P^{\frac{1}{3}} + A_T^{\frac{1}{3}}\right), \quad i = V, W, C, \quad (9)$$

Where A_P, A_T and Z_P, Z_T are the mass and charge numbers of deuteron, ^{16}O (entrance channel) and ^{6}Li, ^{12}C (exit channel).

Figure 1 illustrates the OM and CC description of the elastic deuteron scattering on the ^{12}C and ^{16}O targets in comparison with the data measured at 52 MeV [25] and 56 MeV [26]. The Coulomb potentials parameter r_C = *0.95* fm was used in all calculations. The Woods-Saxon (WS) parameters of the OP's of the d +^{12}C and d+^{16}O systems are shown in Table I. For the exit channel, the OP parameters of the ^{6}Li+^{12}C system have been adjusted to reproduce the elastic data measured at $E_{\text{lab.}}$ = 63 MeV [27], corresponding to $E_{\text{c.m.:}}$ = 42.0 MeV that is close to the c.m. energy of the final partition in the ^{16}O(d,^{6}Li)^{12}C reaction.

Another input for the CRC calculation of the ^{16}O(d,^{6}Li)^{12}C reaction is the structure information of ^{6}Li, which is formed by the incident deuteron and α cluster from the target. In this present work, the relative motion of the d and α in the ground state of ^{6}Li is assumed to be in 2S state, and the corresponding spectroscopic amplitude (the overlap of deuteron and ^{6}Li nucleus) is taken to be unity. The binding potential between the deuteron and α-cluster in ^{6}Li is adopted in the WS form with R = 1.905 fm and a = 0.65 fm. The potential depth of 77.5 MeV has been adjusted to reproduce d separation energy E = 1.47 MeV.

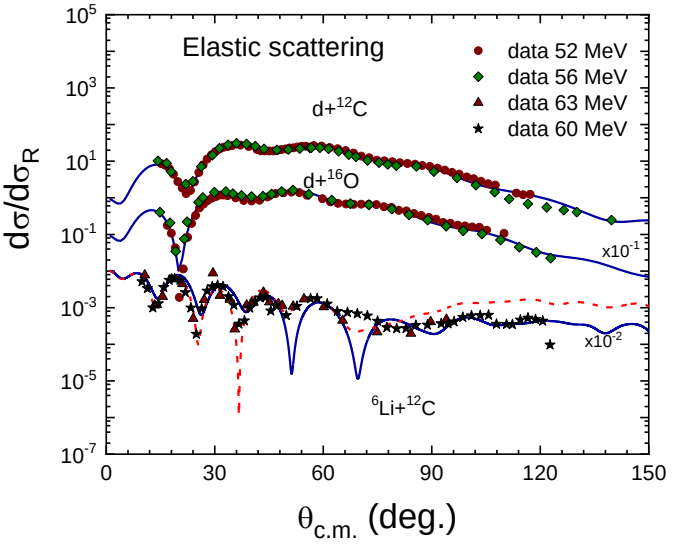


Fig. 1. The OM and CC results for the elastic d+^{12}C , d+^{16}O and ^{6}Li+^{12}C elastic scattering obtained with the OP's assumed in the standard WS form (Table 1), in comparison with the data measured at $E_{\text{lab.}}$ = 52 MeV [25], 56 MeV [26] for the d+^{12}C, d+^{16}O systems, and at $E_{\text{lab.}}$ = 63 MeV [27] and 60 MeV [28] for the ^{6}Li+^{12}C system. The dash-dotted line is the OM result given by the elastic scattering wave function used in the DWBA calculation of the α transfer reaction

In our CRC analysis of the ^{16}O(d,^{6}Li)^{12}C reaction measured at $E_{\text{lab.}}$ = 54.25 MeV [15,20], the OP of the d +^{16}O system E_{lab}. = 52 MeV and ^{6}Li+^{12}C system at $E_{\text{lab.}}$ = 63 MeV are used to generate the (relative) scattering wave functions, $\chi_{d+^{16}O}$ and $\chi_{^{6}Li+^{12}C}$, respectively. The binding potential $V_{12C-\alpha}$ of the α-cluster

and ^{12}C in ^{16}O is taken in the WS form with fixed geometry (as R = 4.148 fm, a = 0.55 fm in CRC calculations of the direct α transfer, and R = 3,683 fm, a = 0.55 fm in CRC calculations of the indirect α transfer [29]), and the potential depths were adjusted to reproduce α separation energy of 7.162 MeV. The OP for the ^{6}Li+^{12}C system at 63 MeV is also used in the transfer interaction W in equation (2). The OP of the d +^{12}C system chosen to reproduce the elastic scattering data at $E_{lab.}$ = 52 MeV [25] is used for the core-core OP in the transfer interaction. Thus, all the necessary physics inputs for the CRC calculation are properly chosen, and only the spectroscopic factors of the α cluster in ^{16}O that characterize the overlap of the intrinsic wave functions of ^{12}C and ^{16}O remain the free parameters.

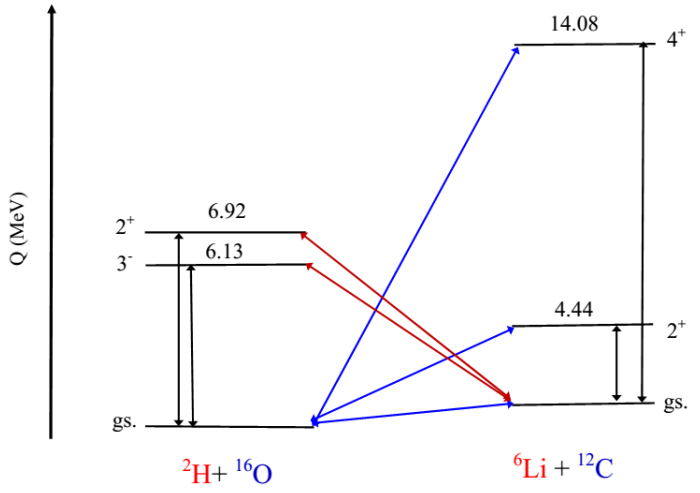


Fig. 2. Coupling scheme of the six reaction channels taken into account in the CRC calculations of the α transfer ^{16}O(d,^{6}Li)^{12}C reaction, which includes both the direct and indirect α transfer processes

The present CRC caclculation includes also the indirect α transfer process, with the ^{16}O target being excited to the 3^-_1 (6.13 MeV) and 2^+_1 (6.92 MeV) excited states before the α transfer. The α spectroscopic factors of the excited ^{16}O were taken from shell model results, with S_α = 0.663 and 0.5 for the 3^-_1 and 2^+_1 states, respectively [10]. Transition potentials between the ground state and excited states of ^{16}O were determined by deforming the OP using the deformation lengths $\delta_{2^+} = 1.025$ fm and $\delta_{3^-} = 1.831$ fm that correspond to the electric transition probabitilies $B(E2) = 39.3\ e^2$ fm^4 and $B(E3) = 1490\ e^2$ fm^6, respectively. The detailed coupling scheme of the present CRC calculations of the α transfer reaction is shown in Fig. 2. The binding potentials $V_{12C-\alpha}$ between the α-particle and ^{12}C core in the excited ^{16}O nucleus were taken the same as that used for ^{16}O in the ground state.

The obtained CRC results are compared with experimental data [15] in Fig. 3, and the agreement between the calculated cross sections with the data is quite reasonable for the 3 observed states of ^{12}C. We note that the same ^{6}Li+^{12}C optical potential has been used in the CRC calculation 3 exit channels with ^{12}C being in the g.s. and excited 2^+ and 4^+ states. Without inclusion of the indirect α transfer processes, the α spectroscopic factors S_α = 1.960, 1.756, and 0.731 were deduced for the ground, 2^+ and 4^+ states, respectively. We remark that the α spectroscopic factor S_α (0^+) = 1.960 for the

ground state of ^{12}C deduced from the present direct CRC calculations is same those taken in the direct CRC calculations of the elastic α transfer [29]. The direct and indirect CRC results are illustrated as the solid lines in Figure 3, with all OP parameters unchanged. Then, the α spectroscopic factors S_α = 0.715, 3.90 and 0.723 were deduced for the ground, 2^+ and 4^+ states, respectively. We note that the CRC calculations of the indirect α transfer include not only the contributions of the excited states of ^{16}O, but also contribution of the excited states of ^{12}C in the exit channel. We have found that while the large α spectroscopic factors of ^{16}O for the ground state obtained from the CRC results without the indirect transfer decrease significantly when the indirect transfer via the excited states of ^{16}O and ^{12}C included, those for the 2^+ state of ^{12}C increase to be a factor of two times. These values of the α spectroscopic factors of ^{16}O again are similar those deduced from the direct and indirect CRC analysis of the elastic α transfer reation at large angle in the elastic ^{12}C(^{16}O, ^{16}O)^{12}C scattering [29]. Although the DWBA results shown in Fig. 3 using the α spectroscopic factors S_α = 0.43, 2.34, and 4.0 describe well the experimental data [20], the OP's used for the entrance and exit channels are not appropriate (the OP for the entrance channel cannot describe the elastic deuteron scattering on ^{16}O at 52 MeV, while the ^{6}Li+^{12}C optical potential at 50.6 MeV [20, 30] is chosen for exit channel). These results show that the OP plays a vital role in the CRC analysis of the α transfer reaction.

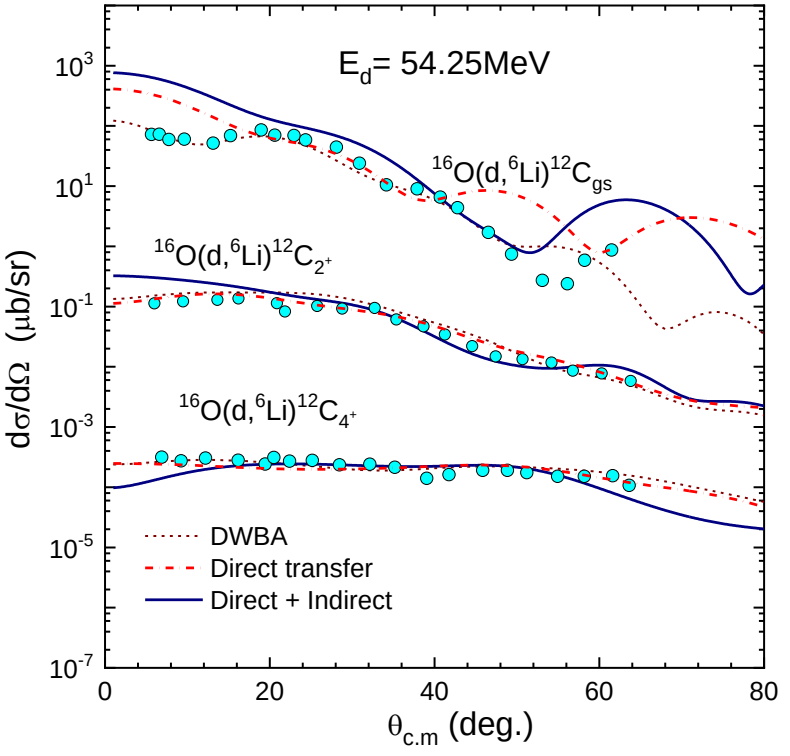


Fig. 3. CRC results for the α transfer ^{16}O(d,^{6}Li)^{12}C reaction in comparison with experimental data measured at E_α = 54.25 MeV [15, 20]. Dash lines are the DWBA results using the spectroscopic factors and OP taken

from Refs. [15, 20]. The dash-dotted lines are the CRC results not including the indirect transfer processes, and the solid lines are those with the indirect transfer processes included

IV. CONCLUSIONS

The data of the α transfer ^{16}O(d,^{6}Li)^{12}C* reaction measured at E_d = 54.25 MeV [15,20] have been analyzed using the CRC method, including both the direct and indirect α transfer processes. The optical potentials for the entrance and exit channels as well as the core-core OP were obtained accurately from the OM and CC fits to the elastic scattering data of the d +^{16}O, d +^{12}C systems at E_{lab}. = 52 MeV and the ^{6}Li+^{12}C system at 63 MeV and 60 MeV. The α spectroscopic factors of ^{16}O deduced from the full CRC calculation, with both the direct and indirect transfer processes included, are significantly changed those deduced from the CRC calculation that included only the direct transfer process. These spectroscopic factors show a strong impact of the channel coupling to the excited states of both ^{16}O and ^{12}C, especially, the 2^{+} state of ^{12}C with a quite large α spectroscopic factor $S_{\alpha}(2^{+})$ = 3.90. The best-fit α spectroscopic factors deduced from the present CRC analysis are close to the S_{α} values reduced from the systematic CRC analysis of the elastic α transfer reation to contribute to the large angle of the elastic ^{12}C(^{16}O, ^{16}O)^{12}C scattering in energy range of 100-300 MeV [29]. The present work has shown also the important role by the OP chosen for the exit channel in the CRC calculation of the α transfer reaction.

ACKNOWLEDGEMENT

The present research has been supported by the National Foundation for Scientific and Technological Development (NAFOSTED Project No. 103.04-2016.35).

REFERENCES

[1]. Y. Kanada-En'yo, Prog. Theor. Phy. 117, 655, 2007.

[2]. C. Kurokawa and K. Kato, Nucl. Phys. A 792, 87, 2007.

[3]. Y. Funaki, H. Horiuchi, W. von Oertzen, G. R¨opke, P. Schuck, A. Tohsaki, and T. Yamada, Phys. Rev. C 80, 064326, 2009.

[4]. T. Yamada, Y. Funaki, T. Myo, H. Horiuchi, K. Ikeda, G. R¨opke, P. Schuck, and A. Tohsaki, Phys. Rev. C 85, 034315, 2012.

[5]. Y. Suzuki, Progress of Theoretical Physics 55, 1751, 1976, https://academic.oup.com/ptp/article-pdf/55/6/1751/5264825/55-6-1751.pdf.

[6]. Y. Suzuki, Progress of Theoretical Physics 56, 111, 1976, https://academic.oup.com/ptp/article-pdf/56/1/111/5430067/56-1-111.pdf.

[7]. D. Kurath, Phys. Rev. C 7, 1390, 1973.

[8]. Rotter, Fortschritte der Physik 16, 195, 1968, https://onlinelibrary.wiley.com/doi/pdf/10.1002/prop.196.

[9]. M. Ichimura, A. Arima, E. Halbert, and T. Terasawa, Nuclear Physics A 204, 225, 1973.

[10]. Volya and Y. M. Tchuvil'sky, Phys. Rev. C 91, 044319, 2015.

[11]. C. Dtraz, H. Duhm, and H. Hafner, Nuclear Physics A 147, 488, 1970.

[12].F. Becchetti, K. Hecht, J. Jnecke, D. Overway, and G. Kekelis, Nuclear Physics A 339, 132, 1980.

[13].G. J. Wozniak, D. P. Stahel, J. Cerny, and N. A. Jelley, Phys. Rev. C 14, 815, 1976.

[14].W. Steele, P. Smith, J. Finck, and G. Crawley, Nuclear Physics A 266, 424, 1976.

[15].T. Yamaya, K. Umeda, T. Suehiro, K. Takimoto, R. Wada, E. Takada, M. Fukada, J. Schimizu, and Y. Okuma, Physics Letters B 90, 219, 1980.

[16].C. W. Wang, N. S. Chant, P. G. Roos, A. Nadasen, and T. A. Carey, Phys. Rev. C 21, 1705, 1980.

[17].J. D. Sherman, D. L. Hendrie, and M. S. Zisman, Phys. Rev. C 13, 20, 1976.

[18].N. S. Chant and P. G. Roos, Phys. Rev. C 15, 57, 1977.

[19].T. A. Carey, P. G. Roos, N. S. Chant, A. Nadasen, and H. L. Chen, Phys. Rev. C 29, 1273, 1984.

[20].K. Umeda, T. Yamaya, T. Suehiro, K. Takimoto, R. Wada, E. Takada, S. Shimoura, A. Sakaguchi, S. Murakami, M. Fukada, and Y. Okuma, Nuclear Physics A 429, 88, 1984.

[21].J. Thompson, Comput. Phys. Rep. 7, 167, 1988, http://www.fresco.org.uk.

[22].G. Satchler, *Direct Nuclear Reactions*, Clarendon Press, Oxford, 1983.

[23].G. F. Steyn, S. V. F¨ortsch, A. A. Cowley, J. J. Lawrie, G. J. Arendse, G. C. Hillhouse, J. V. Pilcher, F. D. Smit, and R. Neveling, Phys. Rev. C 59, 2097, 1999.

[24]. J. Mabiala, A. A. Cowley, S. V. F¨ortsch, E. Z. Buthelezi, R. Neveling, F. D. Smit, G. F. Steyn, and J. J. Van Zyl, Phys. Rev. C 79, 054612, 2009.

[25].F. Hinterberger, G. Mairle, U. Schmidt-Rohr, G. Wagner, and P. Turek, Nuclear Physics A 111, 265, 1968.

[26].N. Matsuoka, H. Sakai, T. Saito, K. Hosono, M. Kondo, H. Ito, K. Hatanaka, T. Ichihara, A. Okihana, K. Imai, K. Nisimura, Nuclear Physics A 455, 413, 1986.

[27].V. A. Maslov, R. A. Astabatyan, I. Vincour, A. S. Denikin, T. K. Zholdybaev, V. I. Zagrebaev, R. Kalpakchieva, I. V. Kuznetsov, S. P. Lobastov, S. M. Lukyanov, E. R. Markaryan, Y. E. Penionzhkevich, N. K. Skobelev, Y. G. Sobolev, V. Y. Ugryumov, and A. A. Hassan, Bulletin of the Russian Academy of Sciences Physics , 404, 2005.

[28].H. G. Bingham, M.L. Halbert, D.C. Hensley, E. Newman, K.W. Kemper, and L.A. Charlton, Phys. Rev. C 11, 1913, 1975.

[29]. Nguyen Tri Toan Phuc, Nguyen Hoang Phuc, and Dao T. Khoa, Phys. Rev. C 98, 024613, 2009.

[30].L. Chua, F. Becchetti, J. Jnecke, and F. Milder, Nuclear Physics A 273, 243, 1976.

[31].W. Oelert, A. Djaloeis, C. Mayer-Bricke, P. Turek, and S. Wiktor, Nuclear Physics A 306, 1, 1978.