



Towards the intermolecular force in charged AdS black hole

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Abstract: In this paper, the intermolecular force of charged AdS black holes (BH) is investigated. The equation of state of BH in the form of the van der Waals (vdW) equation is proposed and it is shown that this equation describes accurately the BH phase transition. Based on the Lennard- Jones (LJ) potential the modified LJ force which is compatible with this vdW equation is established. Of particular interest, for the first time this force can be always written as the sum of the topological force, created by the topological charge, and the electrostatic force, created by charged conducting micro-sphere with a finite radius. Then the phase transition of BH is totally controlled by these forces. In the process of phase transition from small to large BHs the sum of these forces changes from repulsive to attractive forces and this behavior is supported by the scalar curvature of thermodynamics Riemannian geometry [20]. Combining these properties, a physical picture of the BH molecules is shown that: they behave like charged conducting micro-spheres which bear topological charge. It is necessarily to remark that although our intermolecular force is better than the one obtained recently in [29], it remains an approximate force.

Keywords: AdS black holes, topological charge, intermolecular force, electrostatic force, topological force.

I. INTRODUCTION

The thermodynamics of charged AdS BHs was firstly formulated in [1] and, as the next step, the microscopic structures of these BHs and the micro-molecular force between their constituents become hot topics. They have been investigated in various papers, among them we highly appreciate Refs. [2, 3, 4]. A major breakthrough of this trend is to treat the cosmological constant Λ [1] as the thermodynamic pressure P ,

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi L^2}. \quad (1)$$

For simplicity we focus on the four dimensional Reissner–Nordström -AdS black holes (RN-AdS BH) whose metric reads:

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega_{2,k}^2,$$

Where:

$$f(r) = k - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{L^2}. \quad (2)$$

In (2) $d\Omega_{2,k}^2$ is the metric of a two-sphere S^2 of radius $1/\sqrt{k}$, $k > 0$. For $k=0$ and $k < 0$, $d\Omega_{2,k}^2$ represents respectively the metric of a plane and 2-dimensional hyperboloid. The parameters M and Q are related to the

mass and charge of BH by corresponding factors. The horizon is defined as the largest root of the equation:

$$f(r_+) = k - \frac{2M}{r_+} + \frac{Q^2}{r_+^2} + \frac{r_+^2}{L^2} = 0. \quad (3)$$

After (1), (2) and (3) are taken into account, the temperature T and entropy S of BH are respectively given by

$$T = \frac{1}{4\pi} \left(\frac{df(r)}{dr} \right)_{r=r_+} = \frac{k}{4\pi r_+} - \frac{Q^2}{4\pi r_+^3} + 2Pr_+, \quad (4a)$$

$$S = \frac{A}{4} = \pi r_+^2. \quad (4b)$$

In this set up M becomes the enthalpy of the system. If we interpret k as a new measure of charge, in the context of topological charge [5, 6], the extended first law of black hole thermodynamics [7, 8] yields the following result:

$$dM = TdS + \omega d\varepsilon + VdP + \phi dQ, \quad (5)$$

Here $\varepsilon = 4\pi k$ represents the topological charge, with its conjugate potential being $\omega = \frac{r_+}{8\pi}$. The volume $V = \frac{4\pi r_+^3}{3}$ is conjugate to pressure P , while the charge Q is conjugate to the potential $\phi = \frac{Q}{r_+}$.

Based on (4) and (5) we obtain the BH equation of state:

$$P = \frac{T}{2r_+} - \frac{\varepsilon}{32\pi^2 r_+^2} + \frac{Q^2}{8\pi r_+^4}. \quad (6)$$

The present paper is organized as follows. In Section 2 we establish the modified Lennard – Jones (LJ) potential and consider how it describes the BH phase transition. Section 3 is devoted to the application of

Ruppeiner thermodynamic geometry. The interpretation of BH microscopic structure is presented in Section 4 and the final Section deals with the conclusion and outlook.

II. THE INTERMOLECULAR FORCE IN CHARGED ADS BH

The critical point of the BH transition is obtained from:

$$\left(\frac{\partial P}{\partial v} \right)_{T,\varepsilon,Q} = 0, \quad \left(\frac{\partial^2 P}{\partial v^2} \right)_{T,\varepsilon,Q} = 0,$$

$$v = 2r_+,$$

Which gives:

$$P_c = \frac{\varepsilon^2}{1536\pi^3 Q^2}, T_c = \frac{\varepsilon^{3/2}}{24\sqrt{6}\pi^{5/2} Q}, r_c = \frac{2\sqrt{6}\pi Q}{\sqrt{\varepsilon}}. \quad (7)$$

Eqs. (7) indicates that the BH phase transition takes place only when $\varepsilon > 0$ and $Q > 0$. Therefore, in this paper only positive ε, Q will be considered. It is worth to emphasize that topological charge is specified for BH, it plays a drastic role in the microscopic structure of BH [7, 8] as shown later. It is very interesting to note that from (7) we derive the relation:

$$\frac{P_c v_c}{T_c} = \frac{3}{8}, \quad (8)$$

Which coincides fully with the analogous relation predicted for all fluids, including the vdW fluid. In this regard, the BH equation of state (6) in the form of vdW equation reads:

$$\left(P + \frac{a}{v^2} \right) (v - b) = T, \quad (9a)$$

Where:

$$v = 2r_+, a = \frac{3\varepsilon}{16\pi^2}, b = \frac{4\sqrt{6}\pi Q}{3\sqrt{\varepsilon}}. \quad (9b)$$

Hence, it is realized that the critical values of T, P and v of (9) satisfy (8). According to the law of corresponding states the equations (6) and (9) provide the states in the same class [9]; in the other words, the transition from small to large BHs is of the vdW type [3, 4]. This fact demonstrates that the vdW equation (9) is a very good approximation of the BH equation of state. The aim now is to provide a quantitative proof

of this statement. Equations (6) and (9) are presented below in their respective forms:

$$T_{\text{BH}} = Pv + \frac{\varepsilon}{8\pi^2 v} - \frac{2Q^2}{\pi v^3}, \quad (10)$$

and:

$$T_{\text{vdW}} = P(v - b) + \frac{a}{v} \left(1 - \frac{b}{v} \right). \quad (11)$$

Eqs. (10), (11) are plotted in Figure 1.

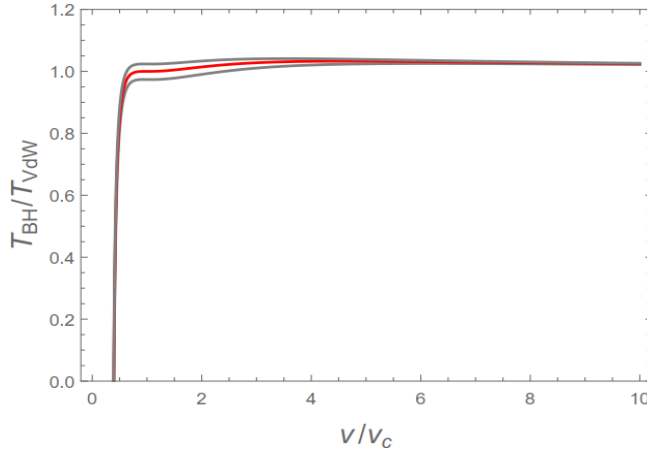


Fig. 1. Comparison of the BH temperature and the temperature of vdW fluid at different values of $\frac{P}{P_c}$.

Figure 1 shows that the vdW fluid describes accurately the charged AdS BH.

The first step is to determine the molecule number N and the molecule density n . They are given in [3, 10]

$$N = \frac{A}{6l_p^2}, n = \frac{N}{V} = \frac{1}{2l_p^2 r_+}, \quad (12)$$

Where the Planck length l_p is determined by $l_p = \sqrt{\hbar G_N / c^3}$.

Based on (6), (7) and (9), we depict the evolution of n/n_c versus $t = T/T_c$ in Fig. 2 which enlightens accurately the small – large BHs transition as follows.

At $t = 0$ the small BH is in frozen state and its number density gets maximum value

$n/n_c = 2.44$ which is a little bit smaller than the corresponding value $n/n_c = 3.0$ in the vdW theory [11]. As t rises, the number density n of small BH monotonously decreases. At the critical temperature $t = 1$, BH undergoes a phase transition from small to large BHs. Finally, at $t = 0$, the reduced number density of large BH vanished, which implied that the mean distance between two constituents tends to infinity.

In order to explore the intermolecular force of the vdW fluid, the Lennard – Jones potential has been considered as the one to control the liquid – gas phase transition [11]. This potential is of the form

$$\phi_{\text{LJ}}(r) = 4\phi_{\text{min}} \left[\left(\frac{r_0}{r} \right)^{12} - \left(\frac{r_0}{r} \right)^6 \right], \quad (13)$$

Where r denotes the separation between two molecules, r_0 is the value at which $\phi_{LJ} = 0$ and the potential attains minimum ϕ_{\min} at $r_{\min} = d = 2^{1/6} r_0$. The quantity d is the diameter of a molecule.

The corresponding interaction force between two adjacent molecules reads

$$F_{LJ}(r) = -\frac{d\phi_{LJ}(r)}{dr} = \frac{24\phi_{\min}}{r} \left[2\left(\frac{r_0}{r}\right)^{12} - \left(\frac{r_0}{r}\right)^6 \right]. \quad (14)$$

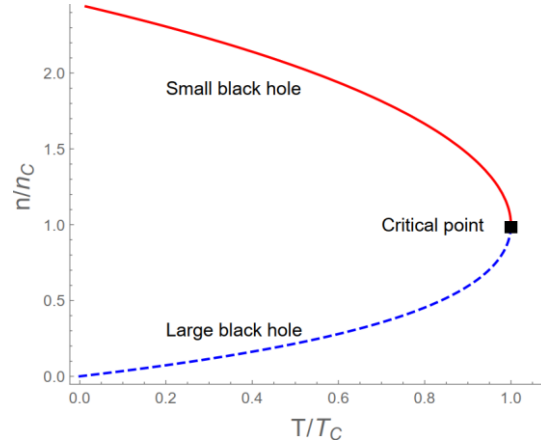


Fig. 2. Phase diagram in the $n/n_c - t$ plane. The BH number density gets maximum value at $t = 0$ and there occurs the phase transition from small to large BH at $t = 1$.

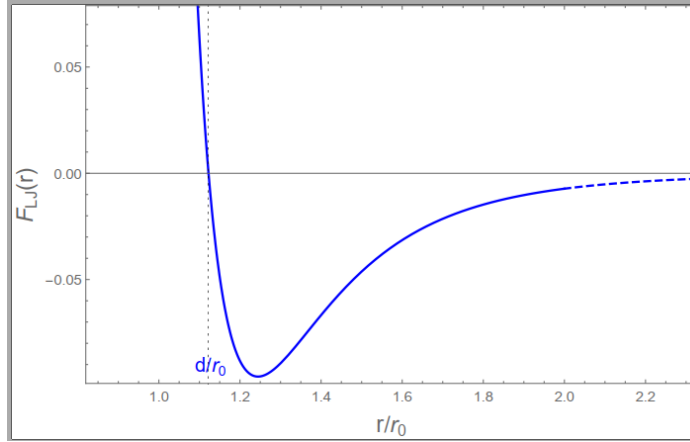


Fig. 3. The r dependence of $F_{LJ}(r)$. The solid (dashed) lines correspond to small (large) BH

The graphs of $F_{LJ}(r)$ in Fig. 3 show that: The LJ potential and LJ force are usually chosen for formulating theories of matters (for instance, the soft – matter, solid states and complex substances) in which the model parameters are adjusted consistently [12 – 18]. Due to the similarity between the BH phase transition and the liquid – gas phase transition in the vdW theory, this potential was adopted in

[2] in order to study the behaviors of the LJ force in the BH phase transition.

Combining Fig. 2 and Fig. 3 the following properties of $F_{LJ}(r)$ are obtained in [2] as follows:

- a. The small BH is in the frozen state when $t = 0$. Then the distance between two micro-molecules is minimum $r = r_0$ and $F_{LJ}(r)$ is repulsive, $F_{LJ}(r) > 0$.

b. For t approaching its transmutation value $t = t_0 = 0.758$, the mean distance r between two molecules equals to d , in which $F_{L}(d) = 0$.

c. For t rising from t_0 to 1, the mean distance r is larger than d , $r > d$, the interaction force becomes attractive, $F_{L}(r) < 0$.

In present paper we proceed to highlight the role of the topological and electric charges in the BH phase transition. In order that the potential (13) describes the BH phase transition r_0 and ϕ_{\min} are adjusted as

$$r_0^3 = \frac{b}{\sqrt{2}}, \phi_{\min} = \frac{9a}{10\pi b}. \quad (15)$$

Then we get the modified Lennard – Jones potential and the corresponding modified force which controls the BH phase transition

$$F(r) = \frac{24\phi_{\min}}{r} \left[2 \left(\frac{r_0}{r} \right)^{12} - \left(\frac{r_0}{r} \right)^6 \right], \quad (16)$$

Here $r_0(\varepsilon, Q)$ and $\phi_{\min}(\varepsilon, Q)$ are given in Eq. (15). Eq. (16) tells that $F(r)$ turns out to be the function of r and two types of charges, $F(r) = F(r, \varepsilon, Q)$. In order that this force becomes the interaction force between two adjacent molecules, the topological charge ε

and electric charge Q of BH must be replaced by the corresponding charges of molecules:

$$\varepsilon_0 = \frac{\varepsilon}{N}, q = \frac{Q}{N},$$

Where N is the number of molecules determined in (12). As a result, the interaction force between the BH molecules reads

$$F_{\text{mol}}(r) = \frac{24\phi_{\min}(\varepsilon_0, q)}{r} \left[2 \left(\frac{r_0(\varepsilon_0, q)}{r} \right)^{12} - \left(\frac{r_0(\varepsilon_0, q)}{r} \right)^6 \right] \quad (17)$$

The interaction force (17) possesses great defect because it does not show up separately the topological force and electric force. To address this issue, one possible solution is to perform several manipulations which ultimately lead Equation (17) to

$$F_{\text{mol}}(r) = b [f_{\varepsilon}(r, \varepsilon_0) + f_Q(r, q)], \quad (18a)$$

Where

$$f_{\varepsilon}(r, \varepsilon_0) = -\frac{27}{40\pi^3} \frac{\varepsilon_0}{r^7}, \quad (18b)$$

is the topological force, created by topological charge, and

$$f_Q(r, q) = \frac{36}{5\pi^2} \frac{q^2}{r^{13}}, \quad (18c)$$

denotes the electrostatic force created by charged conducting micro sphere with radius d and electric charge q [19]. In Fig. 4 are shown the r dependence of $F_{\text{mol}}(r, \varepsilon_0, q)$ at several values of ε_0 and q .

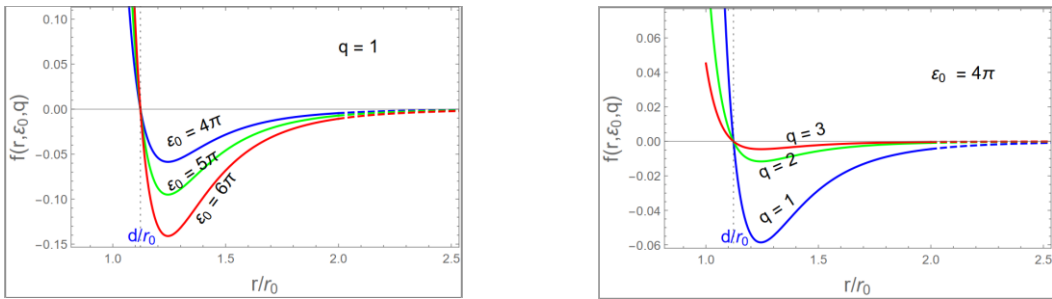


Fig. 4. The evolution of $f(r, \varepsilon_0, q)$ versus r/r_0 at several values of ε_0 and q . The solid (dashed) lines correspond to small (large) BH

In the next section it is proved that the previous properties of intermolecular force (18) will be supported by the Ruppeiner thermodynamic geometry [20].

III. THERMODYNAMIC GEOMETRY

In recent years the thermodynamic geometry has been more and more invoked to study the microscopic structure of charged AdS BHs [2, 3, 21-25]. Because the thermodynamic scalar curvature R associated with BHs provides us with essential information related to the characters of the intermolecular force [26-28]:

- Positive R , $R > 0$, corresponds to a repulsive interaction.

- Negative R , $R < 0$, corresponds to an attractive interaction.

- Vanishing R , $R = 0$, means that there is no interaction.

In order to improve this subject, let us begin with the Riemann space given by the metric:

$$ds^2 = \frac{\partial^2 M}{\partial x^i \partial x^k} dx^i dx^k. \quad (19)$$

By selecting the temperature T and thermodynamic volume v as fluctuation coordinates, and choosing appropriate values for $x_0 = v$ and $x_1 = P$, we can obtain the scalar curvature R .

$$\frac{R}{R_c} = \frac{1 - 3r_+^2}{2r_+^5 t},$$

with

$$R_c = \frac{\varepsilon}{48\pi^2 Q^2}.$$

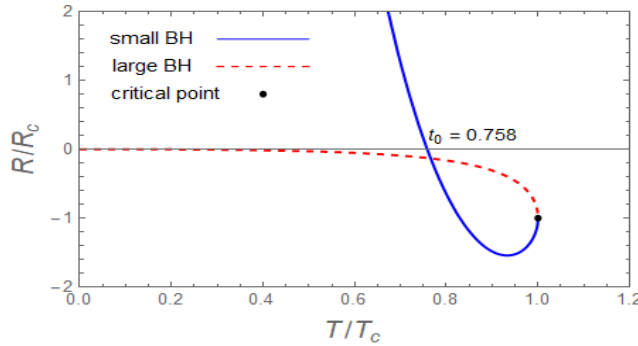


Fig. 5. The evolution of R/R_c versus t . The solid (dashed) lines correspond to small (large) BH

From Figs. 5 it is evident that:

A- In the domain of small BH

- At arbitrary value \bar{t} in the interval $[0, 0.758)$,

$$R(\bar{t}) > 0.$$

- At $\bar{t} = t_0 = 0.758$,

$$R(0.758) = 0. \quad (20)$$

- When \bar{t} belongs to the intervals $(0.758, 1)$

$$R(\bar{t}) < 0.$$

B- In the domain of large BH

- When \bar{t} belongs to the intervals $(0, 1)$

$$R(\bar{t}) < 0.$$

The properties (20) for R are written in geometric language, they are now translated to the physical language in term of the interaction forces taken at different values of topological charge and electric charge. Combining Fig. 2 and Fig. 5, we have:

A- In the domain of small BH

- At arbitrary value \bar{t} in the interval $(0, 0.758)$ the mean distance between BH molecules $\bar{r} < d$ the interaction force $f(\bar{r}) \equiv f(\bar{r}, \varepsilon_0, q)$ obeys the inequalities

$$f(\bar{r}) > 0.$$

- At $\bar{t} = 0.758$,

$$f(d) = 0. \quad (21)$$

- When \bar{t} belongs to the intervals $(0.758, 1)$

$$f(\bar{r}) < 0.$$

B- In the domain of large BH

- For \bar{t} belongs to the intervals $(0.758, 1)$

$$f(\bar{r}) < 0.$$

It is clear that (21) reflects exactly the property of the interaction force (18).

To extend this result, the behaviors of R is considered at different values of ε_0 and q . It is witnessed that the full agreement between Fig. 4 and Fig. 6.

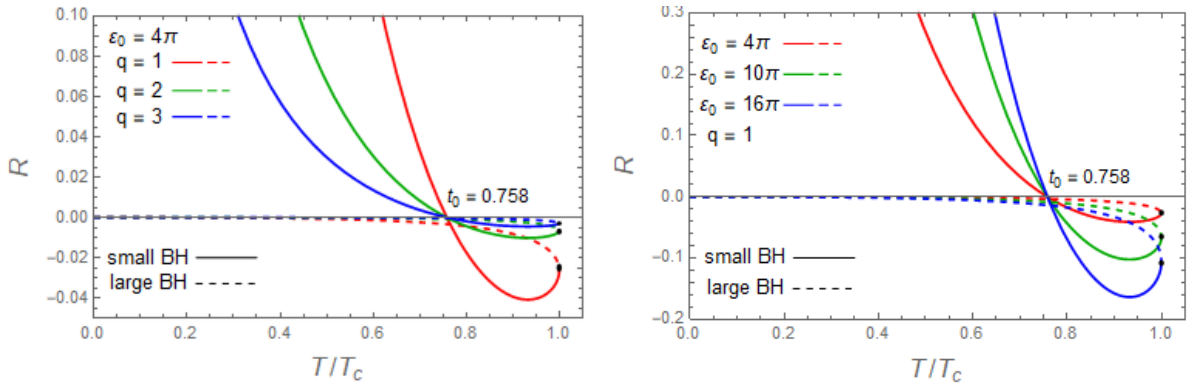


Fig. 6. The evolution of R versus $t = T/T_c$ at several values of ε_0 and q . The solid (dashed) lines correspond to small (large) BH

IV. MICROMOLECULES OF BH

The interaction force (18) is the milestone for us to make clear the structure of the BH micromolecules: they possess the structure of charged conducting microspheres with radius d and their intermolecular forces are determined by (18). The phase transition from small to large BHs is formulated as follows:

a) At $T < T_c$ the small BH behaves like fluid of topological ions.

b) At $T > T_c$ there occurs the phase transition from fluid of topological ions to gas of topological ions.

V. CONCLUSION AND OUTLOOK

The main results obtained in the previous Sections can be summarized as follows.

1) The van der Waals equation is a good approximation of Charged AdS BH.

2) Based on the modified Lennard – Jones potential, the intermolecular force (18) was established,

$$F_{\text{mol}}(r) = b \left[-\frac{27}{40\pi^3} \frac{\varepsilon_0}{r^7} + \frac{36}{5\pi^2} \frac{q^2}{r^{13}} \right],$$

Which consists of two parts:

- The force created by topological charge and.
- The electrostatic force, created by charged conducting micro-spheres with radius d .

3) It is easily seen that our intermolecular force is better than the one obtained in [29].

$$F_{\text{mol}}(r) = \frac{d\phi(r)}{dr},$$

Where:

$$\phi = 4\phi_0 \left[\left(\frac{r_0}{r+r_0} \right)^{12} - \left(\frac{r_0}{r+r_0} \right)^6 \right],$$

$$\phi_0 \approx \frac{0.0003388}{Q}, r_0 = \frac{495(1-4q^2)}{248\pi^2\phi_0}.$$

4) The properties of interaction force were confirmed by the scalar curvature R in the thermodynamic geometry.

5) It is notable that our force is merely an approximate force because we started from the van der Waals equation to find the force.

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