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Phase Transition of Aligned two Higgs Doublets Model in the Cornwall-Jackiw-Tomboulis Formalism

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Abstract: Using the Cornwall-Jackiw-Tomboulis formalism at finite temperature, we consider the phase transition of the aligned two Higgs doublet model in the two-loop double-bubble contribution. The results show that the system obeys a first-order phase transition.

Keywords: *The two-Higgs-doublet model (2HDM), standard model (SM), beyond standard model (BSM), phase transition in early universe*.

I. INTRODUCTION

The discovery of the Higgs boson ten years ago by the Large Hadron Collider (LHC) at the European Council for Nuclear Research (CERN) [1, 2] is a spectacular milestone in particle physics, constituting the last piece of the Standard Model (SM).

The Higgs boson is deeply related to the origin of mass and thermal history of our universe. When the universe was cooling down, at a temperature of order 100 GeV, it went through a transition from a symmetric phase to an electroweak (EW) broken phase, where the Higgs boson(s) condensate and masses of particles generated, i. e. the Higgs field(s) acquired nonvanishing vacuum expectation values (vev). The evolution of this process strongly depends on the shape of the Higgs potential. Distinct profiles for the Higgs potential result in corresponding courses for the electroweak phase transition (EWPT) in the early universe, ranging from the smooth cross-over in the SM, with the observed Higgs mass of 125 GeV [3], to the strong first-order phase transition, with new physics.

Even so, the SM cannot be the ultimate theory of particle physics yet. Despite the undeniable success of the SM in explaining the observed phenomena (in particular regarding the Higgs boson), it is by now clear that there must be physics beyond the SM (BSM). The most striking indications come from inability of the SM to properly explain the baryon asymmetry of the Universe [4], as well as to accommodate both dark matter and neutrino masses [5]. In addition, a new measurement of the muon anomalous magnetic moment has been recently announced by the Fermi National Accelerator Laboratory (FNAL) Muon g-2 collaboration [6], supporting the claim for BSM physics.

The quest for New Physics is thus highly relevant. One of the directions addresses an

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essential question: how many elementary scalar particles are there? After all, there is no fundamental reason why the scalar sector should be restricted to a single Higgs doublet, as predicted by the SM. Moreover, although the neutral scalar observed at the LHC is compatible with the Higgs boson of the SM, it can also correspond to just one particle of a larger set of scalar bosons [4]. The possibility of multi-Higgs bosons is extremely exciting. From the experimental side, searches for additional Higgs bosons have been performed by both A Toroidal LHC Apparatus (ATLAS) [7] and the Compact Muon Solenoid (CMS) [8] collaborations. From the theoretical side, there have been countless studies on models with an extended scalar sector (for reviews, see refs. [9, 10] and references therein). Multi-Higgs Models allow for extremely rich phenomenologies, including for example the possibility of charged scalar bosons.

Particularly appealing inside multi-Higgs scenarios is the simplest one that can provide a new source of charge - parity (CP) violation - as required by the three Sakharov criteria for baryogenesis [11]-, the so-called 2-Higgs-Doublet Model (2HDM) [12] (for a review, see ref. [13]). It became clear that the LHC has not yet detected any significant deviations between the observed properties of the Higgs boson and the SM predictions. Hence, if discrepancies are to be detected, they shall be very subtle. It is therefore of paramount importance that precise predictions from the theory side are put forward, so as to properly interpret small experimental signals of BSM physics. In other words, it is compelling to go beyond the leading-order (LO) predictions of the model and include the next-to-leading-order (NLO).

When higher orders in loop expansion are included, the minimum of the potential is in general modified, so that the true vev no longer corresponds to the tree-level one. It turns out that there is more than one consistent method to select the true vev, in such a way that the different alternatives have significant consequences for the renormalization of the theory. This discussion becomes especially interesting whenever the Goldstone theorem included. The tandem composed of these two topics - two-loop corrections and Goldstone requirement - constitutes another major pillar of this paper. Given the complexity of NLO and irregular integrals, the use of computational tools by dimensional regulation is today virtually indispensable.

Although this 2HDM bring many successful results for nuclear matter and dark matter, the full picture about its phase transition and phase structure is still not clearly [14-19]. In this paper, we considered the aligned 2HDM and it phase structure in the two-loop double-bubble contribution by applying the Cornwall-Jackiw-Tomboulis (CJT) effective action at finite temperature. The gap and Schwinger-Dyson equations of state is used for numerical calculation to find out the order of phase transition in this model and know more about their phase structure.

This paper is organized as follows. In the next section, we present the model of 2HDM. The alignment limit is given in Sec. III. The CJT formulism at finite temperature using for this model considered in Sec. IV. The numeric results and discussions in Sec. V. The last section is conclusion.

II. THE MODEL

In this section, we use the aligned 2HDM model involves two SU(2)L Higgs doublets $\Phi 1$ and $\Phi 2$, both carrying hypercharge $+½$. For simplicity, we make two common assumptions for the scalar potential. The first assumption is that CP is conserved in the scalar sector, leading to only real coefficients. The second one is that there is a Z2 symmetry Φ1 → −Φ1. or Φ2 → −Φ2 forbidding quartic terms that are odd in either Φ1 or Φ2, but such a symmetry can be softly broken by quadratic terms.

The Lagrangian of this model is described by

$$L = (\partial^\mu \Phi_1)^+ (\partial_\mu \Phi_1) + (\partial^\mu \Phi_2)^+ (\partial_\mu \Phi_2) - V, \quad (1)$$

V is the scalar potential. Under these assumptions, the general terms in the scalar potential constructed with Φ1 and Φ2 are given by

$$V = m_1^2 \Phi_1^+ \Phi_1 + m_2^2 \Phi_2^+ \Phi_2 - m_{12}^2 (\Phi_1^+ \Phi_2 + \Phi_2^+ \Phi_1) + \frac{\lambda_1}{2} (\Phi_1^+ \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^+ \Phi_2)^2 - \lambda_3 (\Phi_1^+ \Phi_1)(\Phi_2^+ \Phi_2) + \lambda_4 (\Phi_1^+ \Phi_2)(\Phi_2^+ \Phi_1) + \frac{\lambda_5}{2} [(\Phi_1^+ \Phi_2)^2 + (\Phi_2^+ \Phi_1)^2], \quad (2)$$

Where

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_1 + i\eta_1 \\ \varsigma_1 + u_1 + i\chi_1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_2 + i\eta_2 \\ \varsigma_2 + u_2 + i\chi_2 \end{pmatrix}, \quad (3)$$

With

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ u_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ u_2 \end{pmatrix}. \quad (4)$$

Note that the vevs at zero temperature are denoted by

$$\mathrm{v_i} = \mathrm{u_i}|_{T=0} \quad (5)$$

and the angle β is the ratio of u_1 and u_2:

$$t_\beta = \tan\beta = \frac{u_2}{u_1}. \quad (6)$$

vev is defined by u in which

$$u^2 = u_1^2 + u_2^2, \quad (7)$$

Where for $T = 0$, $u = v \approx 246$ GeV is the Higgs vev in the SM.

The potential V in the tree-level at the electroweak vacuum is

$$V_0 = \frac{m_1^2}{2} u_1^2 + \frac{m_2^2}{2} u_2^2 - m_{12}^2 u_1 u_2 + \frac{\lambda_1}{8} u_1^4 + \frac{\lambda_2}{8} u_2^4 + \frac{(\lambda_3+\lambda_4+\lambda_5)}{5} u_1^2 u_2^2 \quad (8)$$

Since the basic state requiring V to be minimized at the electroweak vacuum, it yields,

$$m_1^2 + \frac{\lambda_1}{2} u_1^2 + \frac{\lambda_3+\lambda_4+\lambda_5}{2} u_2^2 = m_{12}^2 t_\beta,$$
$$m_2^2 + \frac{\lambda_2}{2} u_2^2 + \frac{\lambda_3+\lambda_4+\lambda_5}{2} u_1^2 = m_{12}^2 t_\beta^{-1}. \quad (9)$$

Eqs. (9) are gap equations which define the true vevs u_1 and u_2 at tree-level.

III. THE ALIGNMENT LIMIT

We first using the field translation (3) and inputting to terms of (1) and (2), using (9) leads to

$$L_{\mathrm{mass}} = -\frac{1}{2}\left(m_{12}^2 - \frac{(\lambda_4+\lambda_5)u_1u_2}{2}\right)\left[(\rho_1 \quad \rho_2)\begin{pmatrix} u_2/u_1 & -1 \\ -1 & u_1/u_2 \end{pmatrix}\begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} + (\eta_1 \quad \eta_2)\begin{pmatrix} u_2/u_1 & -1 \\ -1 & u_1/u_2 \end{pmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}\right]$$
$$-\frac{1}{2}(\varsigma_1 \quad \varsigma_2)\begin{pmatrix} \frac{m_{12}^2 u_2}{u_1} + \lambda_1 u_1^2 & -m_{12}^2 + (\lambda_3+\lambda_4+\lambda_5)u_1u_2 \\ -m_{12}^2 + (\lambda_3+\lambda_4+\lambda_5)u_1u_2 & \frac{m_{12}^2 u_1}{u_2} + \lambda_2 u_2^2 \end{pmatrix}\begin{pmatrix} \varsigma_1 \\ \varsigma_2 \end{pmatrix}$$
$$-\frac{1}{2}(m_{12}^2 - \lambda_5 u_1 u_2)(\chi_1 \quad \chi_2)\begin{pmatrix} u_2/u_1 & -1 \\ -1 & u_1/u_2 \end{pmatrix}\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}. \quad (10)$$

Now, we rotate the Two Higgs Doublets Φ_1 and Φ_2 into the Higgs basis

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = R_\beta \begin{pmatrix} \Phi_h \\ \Phi_H \end{pmatrix}, \quad R_\beta = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix}, \quad (11)$$

Where $c_\beta = \cos\beta$, $s_\beta = \sin\beta$.

Then

$$\Phi_h = \begin{pmatrix} G^+ \\ (h + u + iG_0)/\sqrt{2} \end{pmatrix}, \quad \Phi_H = \begin{pmatrix} H^+ \\ (H_0 + iA)/\sqrt{2} \end{pmatrix}, \quad (12)$$

Where $u^2 = u_1^2 + u_2^2$, $u_1 = uc_\beta$; $u_2 = us_\beta$, and

$$G^\pm = (G_1 \pm iG_2)/\sqrt{2}; \quad H^\pm = (H_1 \pm iH_2)/\sqrt{2}. \quad (13)$$

By using (11) we get

$$\begin{pmatrix}\rho_1\\ \rho_2\end{pmatrix} = R_\beta \begin{pmatrix}G_1\\ H_2\end{pmatrix}, \qquad \begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} = R_\beta \begin{pmatrix}G_2\\ H_2\end{pmatrix},$$
$$\begin{pmatrix}\chi_1\\ \chi_2\end{pmatrix} = R_\beta \begin{pmatrix}G_0\\ A\end{pmatrix}, \qquad \begin{pmatrix}\varsigma_1\\ \varsigma_2\end{pmatrix} = R_\beta \begin{pmatrix}h\\ H_0\end{pmatrix}. \qquad (14)$$

Hence (10) leads

$$L_{\text{mass}} = -\left(\frac{m_{12}^2}{c_\beta s_\beta} - \frac{(\lambda_4+\lambda_5)u^2}{2}\right)\begin{pmatrix}G^- & H^-\end{pmatrix}\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}G^+\\ H^+\end{pmatrix}$$
$$-\frac{1}{2}\begin{pmatrix}h & H_0\end{pmatrix}\begin{pmatrix}\lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + 2(\lambda_3+\lambda_4+\lambda_5)c_\beta^2 s_\beta^2 & -c_\beta s_\beta\left[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - (\lambda_3+\lambda_4+\lambda_5)\left(c_\beta^2 - s_\beta^2\right)\right]\\ -c_\beta s_\beta\left[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - (\lambda_3+\lambda_4+\lambda_5)\left(c_\beta^2 - s_\beta^2\right)\right] & \frac{m_{12}^2}{c_\beta s_\beta} + [\lambda_1+\lambda_2-2(\lambda_3+\lambda_4+\lambda_5)]u^2 c_\beta^2 s_\beta^2\end{pmatrix}\begin{pmatrix}h\\ H_0\end{pmatrix}$$
$$-\frac{1}{2}\left(\frac{m_{12}^2}{c_\beta s_\beta} - \lambda_5 u^2\right)\begin{pmatrix}G_0 & A\end{pmatrix}\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}G_0\\ A\end{pmatrix} \qquad (15)$$

Thus, G^0 and $G^\pm$ are Goldstone bosons. In order to prevent $h - H^0$ mixings, the off-diagonal terms of 2 × 2 matrix in (15) should be absent, corresponding to

$$\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 = (\lambda_3+\lambda_4+\lambda_5)\left(c_\beta^2 - s_\beta^2\right). \qquad (16)$$

This is the alignment condition without decoupling in the model. When this condition is satisfied, the tree-level couplings of h to SM particles are exactly the same as those of the SM Higgs boson. Consequently, the tree-level interactions of the CP-even scalar h with weak gauge bosons and SM fermions are totally identical to those of the Higgs boson in the SM. Therefore, the alignment limit means that h does not mix with H^0. In the Higgs basis, the potential terms (2) transform to

$$V = \tilde{m}_1^2 \Phi_h^+\Phi_h + \tilde{m}_2^2 \Phi_H^+\Phi_H - \tilde{m}_{12}^2(\Phi_h^+\Phi_H + \Phi_H^+\Phi_h)$$
$$+\frac{\tilde{\lambda}_1}{2}(\Phi_h^+\Phi_h)^2 + \frac{\tilde{\lambda}_2}{2}(\Phi_H^+\Phi_H)^2 + \tilde{\lambda}_3(\Phi_h^+\Phi_h)(\Phi_H^+\Phi_H)$$
$$+\tilde{\lambda}_4(\Phi_h^+\Phi_H)(\Phi_H^+\Phi_h) + \frac{\tilde{\lambda}_5}{2}[(\Phi_h^+\Phi_H)^2 + (\Phi_H^+\Phi_h)^2]$$
$$+\tilde{\lambda}_6(\Phi_h^+\Phi_H + \Phi_H^+\Phi_h)\Phi_h^+\Phi_h$$
$$+\tilde{\lambda}_7(\Phi_h^+\Phi_H + \Phi_H^+\Phi_h)\Phi_H^+\Phi_H, \qquad (17)$$

Where the new parameters are related to the previous parameters by

$$\tilde{m}_1^2 = m_1^2 c_\beta^2 + m_2^2 s_\beta^2 - 2m_{12}^2 s_\beta c_\beta,$$
$$\tilde{m}_2^2 = m_1^2 s_\beta^2 + m_2^2 c_\beta^2 + 2m_{12}^2 s_\beta c_\beta,$$
$$\tilde{m}_{12}^2 = (m_1^2 - m_2^2)s_\beta c_\beta + m_{12}^2\left(c_\beta^2 - s_\beta^2\right),$$
$$\tilde{\lambda}_1 = \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + 2(\lambda_3+\lambda_4+\lambda_5)s_\beta^2 c_\beta^2,$$
$$\tilde{\lambda}_2 = \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + 2(\lambda_3+\lambda_4+\lambda_5)s_\beta^2 c_\beta^2,$$
$$\tilde{\lambda}_3 = (\lambda_1+\lambda_2-2\lambda_4-2\lambda_5)s_\beta^2 c_\beta^2 + \lambda_3\left(s_\beta^4 + c_\beta^4\right),$$
$$\tilde{\lambda}_4 = (\lambda_1+\lambda_2-2\lambda_3-2\lambda_5)s_\beta^2 c_\beta^2 + \lambda_4\left(s_\beta^4 + c_\beta^4\right),$$
$$\tilde{\lambda}_5 = (\lambda_1+\lambda_2-2\lambda_3-2\lambda_4)s_\beta^2 c_\beta^2 + \lambda_5\left(s_\beta^4 + c_\beta^4\right),$$
$$\tilde{\lambda}_6 = -\left(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2\right)s_\beta c_\beta + (\lambda_3+\lambda_4+\lambda_5)s_\beta c_\beta\left(c_\beta^2 - s_\beta^2\right),$$
$$\tilde{\lambda}_7 = -\left(\lambda_1 s_\beta^2 - \lambda_2 c_\beta^2\right)s_\beta c_\beta + (\lambda_3+\lambda_4+\lambda_5)s_\beta c_\beta\left(s_\beta^2 - c_\beta^2\right) \qquad (18)$$

We input (13) to terms of (17) and (18) leads to the Lagrangian of the system can be decomposed into

$$L = -V_0 + L_{kin} + L_{\text{mass}} + L_3 + L_4, \qquad (19)$$

in which

$$V_0 = \frac{\tilde{m}_1^2}{2}u^2 + \frac{\tilde{\lambda}_1}{8}u^4, \qquad (20)$$

From the minimum condition of V_0 to ensure the basic state of the system we get

$$\tilde{m}_1^2 + \frac{\tilde{\lambda}_1 u^4}{8} = 0, \qquad (21)$$

$$L_{\text{mass}} = -\frac{1}{2}\left(\tilde{m}_1^2 + \frac{\tilde{\lambda}_1}{2}u^2\right)(2G^+G^- + G_0^2) \quad -\left(\tilde{m}_2^2 + \frac{\tilde{\lambda}_3}{2}u^2\right)H^+H^- \quad -\frac{1}{2}\left(\tilde{m}_2^2 + \frac{\tilde{\lambda}_3+\tilde{\lambda}_4-\tilde{\lambda}_5}{2}u^2\right)A^2 \quad -\frac{1}{2}\left(\tilde{m}_1^2 + \frac{3\tilde{\lambda}_1}{2}u^2\right)h^2$$

$$-\frac{1}{2}\left(\tilde{m}_2^2 + \frac{\tilde{\lambda}_3+\tilde{\lambda}_4+\tilde{\lambda}_5}{2}u^2\right)H_0^2 - \left(-\tilde{m}_{12}^2 + \frac{\tilde{\lambda}_6}{2}u^2\right)(G_1H_1 + G_2H_2 + G_0A) - \left(-\tilde{m}_{12}^2 + \frac{3\tilde{\lambda}_6}{2}u^2\right)hH_0, \quad (22)$$

$$L_3 = -\left(\tilde{m}_1^2 + \frac{\tilde{\lambda}_1u^2}{2}\right)uh - \left(-\tilde{m}_1^2 + \frac{\tilde{\lambda}_6u^2}{2}\right)uH_0 - \frac{u}{2}\left(\tilde{\lambda}_1h + \tilde{\lambda}_6H_0\right)\left(2G^+G^- + G_0^2 + h^2\right)$$

$$-\frac{u}{2}\left(\tilde{\lambda}_3h + \tilde{\lambda}_7H_0\right)(2H^+H^- + H_0^2 + A^2) - \frac{\tilde{\lambda}_4-\tilde{\lambda}_5}{2}uA(G_1H_2 - G_2H_1 - G_0H_0 + Ah)$$

$$-u\left(\frac{\tilde{\lambda}_4+\tilde{\lambda}_5}{2}H_0 + \tilde{\lambda}_6h\right)(G_1H_1 + G_2H_2 + G_0A + hH_0), \quad (23)$$

$$L_4 = -\frac{\tilde{\lambda}_1}{8}\left(2G^+G^- + G_0^2 + h^2\right)^2 - \frac{\tilde{\lambda}_2}{8}(2H^+H^- + H_0^2 + A^2)^2 - \frac{\tilde{\lambda}_3}{4}\left(2G^+G^- + G_0^2 + h^2\right)(2H^+H^- + H_0^2 + A^2)$$

$$-\frac{\tilde{\lambda}_4}{4}\left[\left(G_0^2 + h^2\right)(A^2 + H_0^2) + 4G^+G^-H^+H^-\right] - \frac{\tilde{\lambda}_5}{4}\left[\left(G_0^2 - h^2\right)(A^2 - H_0^2) + (G_1^2 - G_2^2)(H_1^2 - H_2^2)\right]$$

$$-\frac{\tilde{\lambda}_4+\tilde{\lambda}_5}{2}\left[(G_1H_1 + G_2H_2)(G_0A + H_0h)\right] + \frac{\tilde{\lambda}_4-\tilde{\lambda}_5}{2}\left[(G_1H_1 - G_2H_2)(G_0H_0 - Ah)\right] - \tilde{\lambda}_5(G_1G_2H_1H_2 + G_0AH_0h)$$

$$-\frac{\tilde{\lambda}_6}{2}\left(2G^+G^- + G_0^2 + h^2\right)(G_1H_1 + G_2H_2 + G_0A + H_0h) + \frac{\tilde{\lambda}_7}{2}(2H^+H^- + H_0^2 + A^2)(G_1H_1 + G_2H_2 + G_0A + H_0h). \quad (24)$$

For $T = 0$, Eq. (22) gives us

$$\tilde{m}_1^2 + \frac{\tilde{\lambda}_1v^2}{2} = 0;$$

$$\tilde{m}_1^2 + \frac{3\tilde{\lambda}_1v^2}{2} = \tilde{\lambda}_1v^2 = m_h^2;$$

$$\tilde{m}_2^2 + \frac{\tilde{\lambda}_3v^2}{2} = \left(\frac{m_{12}^2}{c_\beta s_\beta} - \frac{(\lambda_4+\lambda_5)v^2}{2}\right) = m_{H^\pm}^2;$$

$$\tilde{m}_2^2 + \frac{\tilde{\lambda}_3+\tilde{\lambda}_4+\tilde{\lambda}_5}{2}v^2 = \frac{m_{12}^2}{c_\beta s_\beta} + [\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4 + \lambda_5)]v^2c_\beta^2s_\beta^2 = m_{H^0}^2;$$

$$\tilde{m}_2^2 + \frac{\tilde{\lambda}_3+\tilde{\lambda}_4-\tilde{\lambda}_5}{2}v^2 = \left(\frac{m_{12}^2}{c_\beta s_\beta} - \lambda_5v^2\right) = m_A^2;$$

$$\tilde{m}_{12}^2 - \frac{\tilde{\lambda}_6v}{2} = 0;$$

$$\tilde{m}_{12}^2 - \frac{3\tilde{\lambda}_6v^2}{2} = \tilde{\lambda}_6v^2. \quad (25)$$

In order to prevent $h - H_0$ mixings, $\tilde{\lambda}_6 = 0$ (then $\tilde{m}_{12}^2 = 0$), i.e. (16) satisfied. Here the convention $m_h \le m_{H_0}$ has been chosen, and the SM limit is recovered.

Eqs. (25) show that $G^\pm$ and G^0 represent the charged and neutral massless Goldstone bosons. m_h and m_{H^0} are the masses of the CP-even Higgs bosons, m_A is the mass of the CP-odd Higgs boson, and $m_{H^\pm}$ are the masses of the charged Higgs bosons, respectively. The parameter t_β is chosen in order to have a measure as to how closely the state h, which in the following plays the role of the discovered Higgs boson at $m_h \approx 125$ GeV, resembles the properties of a SM Higgs boson. In the so-called alignment limit [14, 20-25] $\tilde{\lambda}_6 = 0$, the lowest-order couplings of h to the SM particles are precisely as predicted by the SM.

Hence, instead of the eight parameters in the Higgs potential m_1, m_2, m_{12}, λ_1, … λ_5, a more convenient choice of six parameters is

$$v, \beta, m_h, m_{H^0}, m_A, m_{H^\pm}. \quad (26)$$

Where m_{12} or λ_5 is found at $T = 0$ based on these parameters.

In order to investigate the vast parameter space, we carry out a random scan within the following ranges:

$t_\beta \in (0.8, 25)$; $m_h = 125$ GeV; $v = 246$ GeV;

$m_{H^0} \in (150, 1500)$ GeV; $m_A \in (150; 1500)$ GeV;

$m_{H^\pm} \in (150; 1500)$ GeV.

IV. THE EFFECTIVE POTENTIAL

To study the phase transition in the early universe, we use the loop-corrected effective potential at finite temperature. Based on [26] and following closely [27, 28], the CJT effective potential at finite temperature in the imaginary time formalism

$$t \to -i\tau, \text{ with } 0 \le \tau \le 1/T$$

and note

$$\int \frac{d^4k}{(2\pi)^4} f(k) \to iT \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} f(i\omega_n, \vec{k}) \equiv i \int_T f(k),$$

Where $\omega_n = 2\pi nT$. Then,

$$V^{CJT} = V_0 + \int_T \mathrm{tr}\left\{\ln D_c^{-1}(k) + \frac{1}{2}\ln D_a^{-1}(k) + \frac{1}{2}\ln D_h^{-1}(k)\right\}$$
$$+\int_T \mathrm{tr}\left\{D_{0c}^{-1}(k;u)D_c(k) + \frac{1}{2}D_{0a}^{-1}(k;u)D_a(k) + \frac{1}{2}D_{0h}^{-1}(k;u)D_h(k) - 2\mathbf{I}\right\}$$
$$+\frac{\tilde{\lambda}_1}{8}(2P_{c11} + P_{a11} + P_{h11})^2 + \frac{\tilde{\lambda}_2}{8}(2P_{c22} + P_{a22} + P_{h22})^2 + \frac{\tilde{\lambda}_3}{4}(2P_{c11} + P_{a11} + P_{h11})(2P_{c22} + P_{a22} + P_{h22})$$
$$+\frac{\tilde{\lambda}_4}{4}[(P_{a11} + P_{h11})(P_{a22} + P_{h22}) + 2P_{c11}P_{c22}] + \frac{\tilde{\lambda}_5}{4}(P_{a11} - P_{h11})(P_{a22} - P_{h22}),$$

Where the inverse of tree-level propagators read

$$iD_{0c}^{-1}(k,u_i) = \begin{pmatrix} \omega^2 - \vec{k}^2 - \widetilde{m}_1^2 - \frac{\tilde{\lambda}_1 u^2}{2} & 0 \\ 0 & \omega^2 - \vec{k}^2 - \widetilde{m}_2^2 - \frac{\tilde{\lambda}_3 u^2}{2} \end{pmatrix}, \quad (27)$$

$$iD_{0a}^{-1}(k,u_i) = \begin{pmatrix} \omega^2 - \vec{k}^2 - \widetilde{m}_1^2 - \frac{\tilde{\lambda}_1 u^2}{2} & 0 \\ 0 & \omega^2 - \vec{k}^2 - \widetilde{m}_2^2 - \frac{\tilde{\lambda}_3+\tilde{\lambda}_4-\tilde{\lambda}_5}{2}u^2 \end{pmatrix}, \quad (28)$$

$$iD_{0h}^{-1}(k,u_i) = \begin{pmatrix} \omega^2 - \vec{k}^2 - \widetilde{m}_1^2 - \frac{3\tilde{\lambda}_1 u^2}{2} & 0 \\ 0 & \omega^2 - \vec{k}^2 - \widetilde{m}_2^2 - \frac{\tilde{\lambda}_3+\tilde{\lambda}_4+\tilde{\lambda}_5}{2}u^2 \end{pmatrix}, \quad (29)$$

Which correspond to doublets (14), and below notations are introduced

$$P_{\alpha ij} = \int_T iD_{\alpha ij}(k), \quad \text{where} \quad \alpha = c, a, h; \quad i,j = 1,2.$$

Since the physical requirement

$$\frac{\delta V^{CJT}}{\delta u} = 0; \qquad \frac{\delta V^{CJT}}{\delta D_\alpha} = 0 \quad (30)$$

must be satisfied so it leads to the gap and Schwinger-Dyson (SD) equations:

$$\widetilde{m}_1^2 + \frac{\tilde{\lambda}_1}{2}u^2 + \Xi_u = 0, \quad (31)$$

$$iD_\alpha^{-1}(k) = iD_{0\alpha}^{-1}(k;u) - \Xi_\alpha = 0, \quad (32)$$

Where

$$\Xi_u = \frac{\tilde{\lambda}_1}{2}(2P_{c11} + P_{a11} + 3P_{h11}) + \frac{\tilde{\lambda}_3}{2}(2P_{c22} + P_{a22} + P_{h22}) + \frac{\tilde{\lambda}_4}{2}(P_{a22} + P_{h22}) - \frac{\tilde{\lambda}_5}{2}(P_{a22} - P_{h22}) \quad (33)$$

$$iD_\alpha^{-1}(k) = \begin{pmatrix} \omega^2 - \vec{k}^2 - M_{1\alpha}^2 & 0 \\ 0 & \omega^2 - \vec{k}^2 - M_{2\alpha}^2 \end{pmatrix}, \quad (34)$$

$$\Xi_\alpha = \begin{pmatrix} \Xi_{1\alpha} & 0 \\ 0 & \Xi_{2\alpha} \end{pmatrix},$$

With

$$\Xi_{1c} = \frac{\tilde{\lambda}_1}{2}(2P_{c11} + P_{a11} + P_{h11}) + \frac{\tilde{\lambda}_3}{2}(2P_{c22} + P_{a22} + P_{h22}) + \frac{\tilde{\lambda}_4}{2}P_{c22},$$

$$\Xi_{2c} = \frac{\tilde{\lambda}_2}{2}(2P_{c22} + P_{a22} + P_{h22}) + \frac{\tilde{\lambda}_3}{2}(2P_{c11} + P_{a11} + P_{h11}) + \frac{\tilde{\lambda}_4}{2}P_{c11},$$

$$\Xi_{1a} = \frac{\tilde{\lambda}_1}{2}(2P_{c11} + P_{a11} + P_{h11}) + \frac{\tilde{\lambda}_3}{2}(2P_{c22} + P_{a22} + P_{h22}) + \frac{\tilde{\lambda}_4}{2}(P_{a22} + P_{h22}) + \frac{\tilde{\lambda}_5}{2}(P_{a22} - P_{h22}),$$

$$\Xi_{2a} = \frac{\tilde{\lambda}_2}{2}(2P_{c22} + P_{a22} + P_{h22}) + \frac{\tilde{\lambda}_3}{2}(2P_{c11} + P_{a11} + P_{h11}) + \frac{\tilde{\lambda}_4}{2}(P_{a11} + P_{h11}) + \frac{\tilde{\lambda}_5}{2}(P_{a11} - P_{h11}),$$

$$\Xi_{1h} = \frac{\tilde{\lambda}_1}{2}(2P_{c11} + P_{a11} + P_{h11}) + \frac{\tilde{\lambda}_3}{2}(2P_{c22} + P_{a22} + P_{h22}) + \frac{\tilde{\lambda}_4}{2}(P_{a22} + P_{h22}) - \frac{\tilde{\lambda}_5}{2}(P_{a22} - P_{h22}),$$

$$\Xi_{2h} = \frac{\tilde{\lambda}_2}{2}(2P_{c22} + P_{a22} + P_{h22}) + \frac{\tilde{\lambda}_3}{2}(2P_{c11} + P_{a11} + P_{h11}) + \frac{\tilde{\lambda}_4}{2}(P_{a11} + P_{h11}) - \frac{\tilde{\lambda}_5}{2}(P_{a11} - P_{h11}). \quad (35)$$

The SD equations (32) can be reduced to the following system of equations for $M_{i\alpha}$,

$$M_{1c}^2 = \widetilde{m}_1^2 + \frac{\tilde{\lambda}_1 u^2}{2} + \Xi_{1c}$$

$$M_{2c}^2 = \widetilde{m}_2^2 + \frac{\tilde{\lambda}_3 u^2}{2} + \Xi_{2c}$$

$$M_{1a}^2 = \widetilde{m}_1^2 + \frac{\tilde{\lambda}_1 u^2}{2} + \Xi_{1a}$$

$$M_{2a}^2 = \widetilde{m}_2^2 + \frac{\tilde{\lambda}_3 u^2}{2} + \frac{\tilde{\lambda}_4 u^2}{2} - \frac{\tilde{\lambda}_5 u^2}{2} + \Xi_{2a}$$

$$M_{1h}^2 = \widetilde{m}_1^2 + \frac{3\tilde{\lambda}_1 u^2}{2} + \Xi_{1h}$$

$$M_{2h}^2 = \widetilde{m}_2^2 + \frac{\tilde{\lambda}_3 u^2}{2} + \frac{\tilde{\lambda}_4 u^2}{2} + \frac{\tilde{\lambda}_5 u^2}{2} + \Xi_{2h} \quad (36)$$

For the case $\tilde{\lambda}_4 = \tilde{\lambda}_5 = 0$ then

$$0 = \widetilde{m}_1^2 + \frac{\tilde{\lambda}_1 u^2}{2} + \frac{3\tilde{\lambda}_1}{2}(P_{a11} + P_{h11}) + 2\tilde{\lambda}_3 P_{h22}$$

$$M_{1c}^2 = M_{1a}^2 = \widetilde{m}_1^2 + \frac{\tilde{\lambda}_1 u^2}{2} + \frac{\tilde{\lambda}_1}{2}(3P_{a11} + P_{h11}) + 2\tilde{\lambda}_3 P_{h22}$$

$$M_{1h}^2 = \widetilde{m}_1^2 + \frac{3\tilde{\lambda}_1 u^2}{2} + \frac{\tilde{\lambda}_1}{2}(3P_{a11} + P_{h11}) + 2\tilde{\lambda}_3 P_{h22}$$

$$M_{2c}^2 = M_{2a}^2 = M_{2h}^2 = \widetilde{m}_1^2 + \frac{\tilde{\lambda}_3 u^2}{2} + \frac{\tilde{\lambda}_3}{2}(3P_{a11} + P_{h11}) + 2\tilde{\lambda}_2 P_{h22}. \quad (37)$$

Eliminate *u* from (37), it leads to

$$\tilde{\lambda}_1 u^2 = -2\widetilde{m}_1^2 - 3\tilde{\lambda}_1(P_{a11} + P_{h11}) - 4\tilde{\lambda}_3 P_{h22},$$

$$M_{1c}^2 = M_{1a}^2 = -\tilde{\lambda}_1 P_{h11},$$

$$M_{1h}^2 = -2\widetilde{m}_1^2 - 3\tilde{\lambda}_1 P_{a11} - 4\tilde{\lambda}_1 P_{h11} - 4\tilde{\lambda}_3 P_{h22},$$

$$M_{2c}^2 = M_{2a}^2 = M_{2h}^2 = \widetilde{m}_2^2 - \frac{\tilde{\lambda}_3}{\tilde{\lambda}_1}\widetilde{m}_1^2 - \tilde{\lambda}_3 P_{h11} + 2\frac{\tilde{\lambda}_1\tilde{\lambda}_2 - \tilde{\lambda}_3^2}{\tilde{\lambda}_1}P_{h22}. \quad (38)$$

Solving Eqs. (38) we obtain the evolution of vev and effective masses of Higgs bosons vs temperature. Figures show the restoration of symmetry at a critical temperature.

V. NUMERICAL STUDY

For calculation, we use the following integrals

$$P = \int_T iD(k)$$

$$= T\sum_{n=-\infty}^{\infty}\int\frac{d^3k}{(2\pi)^2}\frac{-1}{\omega^2-E^2} = \int\frac{d^3k}{(2\pi)^2}\frac{1}{E}\left(\frac{1}{2}+n_B\right), \quad (39)$$

$$R = \int_T \mathrm{tr}\ln D^{-1}(k) = \int_T \ln\det D^{-1}(k)$$

$$= T\sum_{n=-\infty}^{\infty}\int\frac{d^3k}{(2\pi)^2}[\ln(\omega^2-E_1^2)+\ln(\omega^2-E_2^2)]$$

$$= \int\frac{d^3k}{(2\pi)^2}[E_1 + 2T\,ln\left(1-e^{-E_1/T}\right) + E_2 + 2T\,ln\left(1-e^{-E_2/T}\right)], \quad (40)$$

Where

$$\omega = i\omega_n = 2\pi i n T, \qquad E^2 = \vec{k}^2 + M^2, \qquad n_B = \frac{1}{e^{E/T}-1}.$$

For $T = 0$, (39) and (40) give

$$P_0 = \frac{(\mu^2)^{3/2-d/2}}{2}\int\frac{d^3k}{(2\pi)^3}\frac{1}{E} = \frac{M^2}{16\pi^2}\left(\ln\frac{M^2}{\mu^2}-1\right),$$

$$R_0 = \frac{(\mu^2)^{3/2-d/2}}{2}\int\frac{d^3k}{(2\pi)^3}E = \frac{M^2}{32\pi^2}\left(\ln\frac{M^2}{\mu^2}-\frac{3}{2}\right), \quad (41)$$

With μ is a scale with mass dimension which needs to be introduced to balance the dimension of the integration measure.

We aim to study the parameter space in the region of t_β from 1 to 20. The mass difference between the heavy Higgs states should be small and their scan intervals are therefore chosen to be identical as follows

$$v = 246\text{ GeV}; \quad t_\beta = 2; \quad m_h = 125\text{ GeV};$$

$$m_A \simeq m_{H^0} \simeq m_{H^\pm} = 200\text{ GeV}. \quad (42)$$

Hence

$\tilde\lambda_1 v^2 = m_h^2;$

$\tilde\lambda_3 v^2 = 2m_{H^\pm}^2 - 2\tilde m_2^2;$

$\tilde\lambda_4 v^2 = 2m_{H^0}^2 + m_A^2 - 2m_{H^\pm}^2 = 0;$

$\tilde\lambda_5 v^2 = m_{H^0}^2 - m_A^2 = 0;$

$\tilde\lambda_6 = 0;$

$\tilde m_1^2 = -m_h^2/2;$

$\tilde m_{12}^2 = 0;$

$m_{12}^2 = m_{H^\pm}^2 c_\beta s_\beta;$

$\tilde m_2^2 = -m_h^2/2 + m_{H^\pm}^2;$

$$\tilde\lambda_2 = \frac{-\tilde\lambda_1\left(c_\beta^8+c_\beta^6 s_\beta^2-12c_\beta^4 s_\beta^4+c_\beta^2 s_\beta^6+s_\beta^8\right)}{c_\beta^2 s_\beta^2\left(5c_\beta^4-2c_\beta^2 s_\beta^2+5s_\beta^4\right)} + \frac{2(\tilde\lambda_3+\tilde\lambda_4+\tilde\lambda_5)\left(c_\beta^8-c_\beta^6 s_\beta^2-c_\beta^2 s_\beta^6+s_\beta^8\right)}{c_\beta^2 s_\beta^2\left(5c_\beta^4-2c_\beta^2 s_\beta^2+5s_\beta^4\right)} \quad (43)$$

Where $\tilde\lambda_2$ obtains from (18) owing to (16).

From gap and SD equations, consequence for $T = 0$, $P_{h110} = 0$. Hence, $\mu = 75{,}82$.

Solving Eqs. (38) and ploting the effective potential (26) we obtain figures Fig. 1, Fig. 2 and Fig. 3.

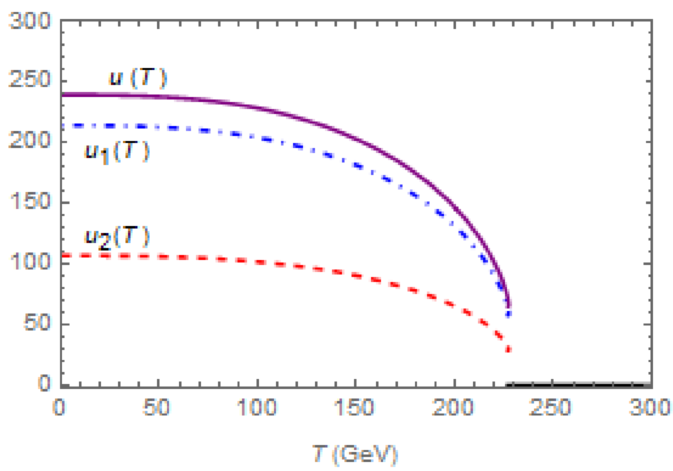

Fig. 1. The evolution of vev u and u1 and u2 vs the temperature

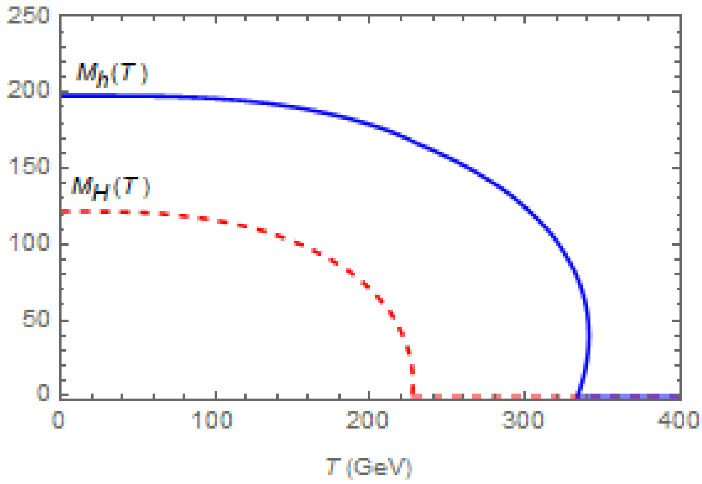

Fig. 2. The evolution of mh and mH vs the temperature

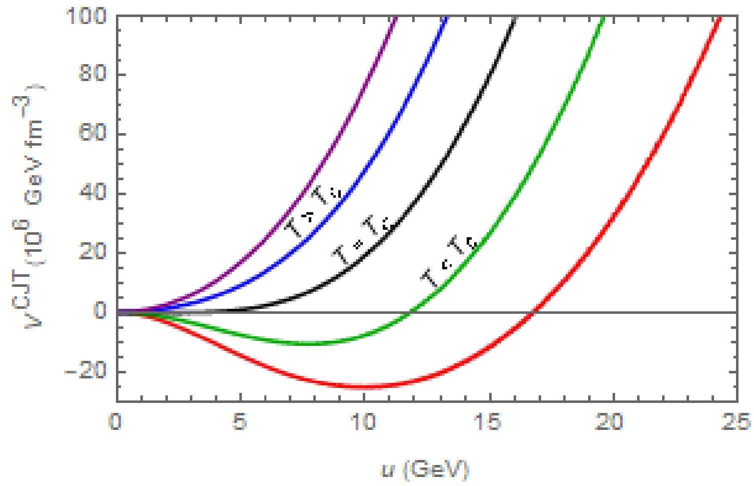

Fig. 3. The evolution of the effective potential vs the vev *u* at various temperature. The temperature increasea from the red line (bottom) to purple line (top)

The figures show that the phase transition is the first order.

VI. CONCLUSIONS

In this paper we have analyzed the thermal history of the 2HDM, and its associated phenomenological imprints by using the CJT formalism in the two-loop double-bubble contribution. It is treated to be an adequate and reliable approach for the study of phase transition [28-30]. Here we are not taking into account the preservation of Goldstone theorem the 2HDM can accommodate a strong first-order electroweak phase transition as showed in Figs. 1 and 2.

Within a simple scenario characterized by the alignment limit without decoupling ($\tilde{\lambda}_6 = 0$) and equal masses for the neutral CP-odd and charged BSM scalars ($m_A \simeq m_{H^0} \simeq m_{H^\pm}$), we have categorized the different thermal histories which are possible of the 2HDM: the Universe undergoes a strong first-order EWPT (from the EW symmetric to the broken phase).

An extension beyond the surveys in this

paper is the general scenario ($\tilde{\lambda}_6 \neq 0$), there we use

$$\begin{pmatrix} h \\ H^0 \end{pmatrix} \to R_\alpha \begin{pmatrix} h \\ H^0 \end{pmatrix},$$

i.e. $$\begin{pmatrix} \varsigma_1 \\ \varsigma_2 \end{pmatrix} = R_{\alpha+\beta} \begin{pmatrix} h \\ H_0 \end{pmatrix}.$$

Here the convention $m_h \leq m_{H^0}$ has been chosen, and the SM limit is recovered when α = 0. This could be in our further research problem.

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REFERENCE

[1]. ATLAS collaboration, *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, Phys. Lett. B716 (2012) 1 [1207.7214].

[2]. CMS collaboration, *Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC*, Phys. Lett. B716 (2012) 30 [1207.7235].

[3]. K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, *Is there a hot electroweak phase transition at m(H) larger or equal to m(W)?*, Phys. Rev. Lett. 77 (1996) 2887 [hep-ph/9605288].

[4]. M. Trodden, *Electroweak baryogenesis: A Brief review*, in 33rd Rencontres de Moriond: Electroweak Interactions and Unied Theories, 1998, [hep-ph/9805252].

[5]. Particle Data Group, P. A. Zyla et al., PTEP 2020, 083C01 (2020).

[6]. Muon g-2, B. Abi et al., Phys. Rev. Lett. 126, 141801 (2021), [2104.03281].

[7]. ATLAS, L. Morvaj, PoS EPS-HEP2019, 584 (2020).

[8]. CMS, J. Tao, PoS LeptonPhoton2019, 182 (2019).

[9]. J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, Front. Phys. 80, 1 (2000).

[10]. I. P. Ivanov, Prog. Part. Nucl. Phys. 95, 160 (2017), [1702.03776].

[11]. A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5, 32 (1967), [Usp. Fiz. Nauk161,61(1991)]

[12]. T. D. Lee, Phys. Rev. D8, 1226 (1973).

[13]. G. C. Branco et al., Phys. Rept. 516, 1 (2012), [1106.0034].

[14]. Lei Wang, Xiao-Fang Han, *Status of the aligned two-Higgs-doublet model confronted with the Higgs data*, [hep-ph/1312.4759].

[15]. Martin Jung, Antonio Pich, Paula Tuz´on, *Charged-Higgs phenomenology in the Aligned two-Higgs-doublet model*, [hep-ph/1006.0470].

[16]. S. Inoue, M.J. Ramsey-Musolf and Y. Zhang, *CP-violating phenomenology of flavor conserving two Higgs doublet models*, Phys. Rev. D 89 (2014) 115023 [1403.4257].

[17]. Dorival Gonçalves, Ajay Kaladharan, Yongcheng Wu, *Electroweak phase transition in the 2HDM: Collider and gravitational wave complementarity*, [hep-ph/2108.05356].

[18]. Zhao Zhang, Chengfeng Cai, Xue-Min Jiang, Yi-Lei Tang, Zhao-Huan Yu and Hong-Hao Zhang (2021), *Phase transition gravitational waves from pseudo-Nambu-Goldstone dark matter and two Higgs doublets*, JHEP 05 (2021)160.

[19]. Thomas Biekötter, Sven Heinemeyer, José Miguel No, María Olalla Olea-Romacho, Georg Weiglein, *The trap in the early Universe: impact on the interplay between gravitational waves and LHC physics in the 2HDM*, [hep-ph/2208.14466].

[20]. S Paula Tuzon´ and Antonio Pich (2010), *The Aligned Two-Higgs-Doublet Model*, arXiv:0293v2[heph-ph] 13 Jan 2010.

[21]. Otto Eberhardt,a Ana Peñuelas Martínezb and Antonio Pich (2021), *Global fits in the Aligned Two-Higgs-Doublet model*, JHEP 05, ArXiv ePrint: 2012.09200.

[22]. M. Carena, I. Low, N. R. Shah, and C. E. M. Wagner, *Impersonating the Standard Model Higgs Boson: Alignment without decoupling*, J. High Energy Phys. 04 (2014) 015.

[23]. P. S. B. Dev and A. Pilaftsis, *Maximally symmetric two Higgs doublet model with natural Standard Model alignment*, J. High Energy Phys. 12 (2014) 024; Erratum, J. High Energy Phys. 11 (2015) E).

[24]. P. S. Bhupal Dev, Apostolos Pilaftsis (2017), *Natural Alignment in the Two Higgs Doublet Model*, IOP Conf. Series: Journal of Physics: Conf. Series 873 (2017) 012008. DOI: 10.1088/1742-6596/873/1/012008.

[25]. Ana Penuelas and Antonio Pich 92017), *Flavour alignment in multi-Higgs-doublet models*, JHEP 12 (2017) 084.

[26]. Cornwall, J. M., Jackiw, R. and Tomboulis (1974), *Effective Action for Composite Operators*, Phys. Rev. D10, 2428.

[27]. Amelino G. and So - Young Pi (1993), *Self-consistent improvement of the finite-temperature effective potential*, Phys. Rev. D47 (1993) 2356.

[28]. Tran Huu Phat, Nguyen Tuan Anh, and Le Viet Hoa, *On the chiral phase transition in the linear sigma model*, Eur. Phys. J. A 19, 359–365 (2004).

[29]. Tran Huu Phat, Le Viet Hoa, Nguyen Tuan Anh, and Nguyen Van Long, *High temperature symmetry nonrestoration and inverse symmetry breaking in the Cornwall-Jackiw-Tomboulis formalism*, Phys. Rev. D76 (2007) 125027.

[30]. Tran Huu Phat, Nguyen Van Long, Nguyen Tuan Anh, and Le Viet Hoa, *Kaon condensation in the linear sigma model at finite density and temperature*, Phys. Rev. D78 (2008) 105016.