Nucl. Sci. and Tech, Vol.4, No. 4 (2014), pp. 38-44

Nuclear Science and Technology

Journal homepage: https://jnst.vn/index.php/nst

Multipole expansion for the electron-nucleus scattering at high energies in the unified electroweak theory

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Abstract: The article presents the multipole expansion for the electron-nucleus scattering cross-section at high energies within framework of the unified electroweak theory. The electroweak currents of the nucleus is expanded into simple components with definite angular momentum, which are called the multipole form factors. The multipole expansion of the cross-section is a consequence of the above expansion. Besides the familiar electromagnetic form factors F_L^X, there are new form factors V_L^X and A_L^X related to weak interactions, corresponding to the vector and axial weak currents. The obtained general expressions are applied to the nucleus ${}_3^6Li$, where the partial form factors are computed in the multiparticle shell model.

I. INTRODUCTION

The method of studying the nuclear structure by electron scattering, or more generally, lepton scattering, is highly effective because it provides most detailed results about the inner structure of the nuclei, especially when the attained electron energy becomes more and more higher.

In order to relate the structure of the nucleus with the scattered electrons, the best way is to expand the scattering amplitude into multipole components, each term corresponds to a definite angular momentum *Lm* (of the interaction carrier). Weigert and Rose [1] were the first to perform a complete expansion when the interaction is purely electromagnetic, and the expansion was improved afterwards by Donnelly and Raskin [2]. Owing to this expansion, every partial amplitude corresponding to each multipole can be calculated in details for nuclei, and they clarify many properties of the nuclei.

At high energies that the electron accelerators reached at present, of order *GeV*, the electron-nucleus scattering must be described by the unified electroweak theory. We shall extent the Weigert-Rose method to expand the scattering amplitude in this case. For simplicity, we consider only the scattering in which the nuclei are unoriented and the electrons are unpolarized.

II. THE ELECTRON-NUCLEUS SCATTERING AT HIGH ENERGIES

We shall consider the scattering of electrons at high energies, of order *GeV*, on the nuclei. In order to apply the Born approximation, the target nuclei are supposed to be light or medium, i.e. they have the charge number *Z* which is fairly less than $1/\alpha = 137$. At this energy scale, in the scattering the electron exchanges a photon γ and an intermediate boson Z^0 with the nucleus, the scattering amplitude is of the form:

https://doi.org/10.53747/jnst.v4i4.242

Received 03 November 2014, accepted 20 March 2015

$$M_{fi} = \frac{4\pi\alpha}{Q^2}\left[\bar{u}'\gamma_\alpha u J_F^\alpha(Q) + \lambda\bar{u}'\gamma_\alpha(g_V + g_A\gamma_5) u J_Z^\alpha(Q)\right] \quad (1)$$

where u is the (spinor) state amplitude of the electron, Q is the 4-momentum transfer of electron (to nucleus), $Q = K - K' = (\omega, \mathbf{q})$, $K = (\varepsilon, \mathbf{k})$ and $K' = (\varepsilon', \mathbf{k}')$ are the 4-momenta of the electron before and after scattering respectively, $J_F^\alpha(Q)$ is the electromagnetic current and $J_Z^\alpha(Q)$ is the weak current of the nucleus,

$$\lambda \equiv -\frac{g^2 Q^2}{16\pi\alpha\left(m_Z^2 - Q^2\right)\cos^2\theta_W},$$

g is the weak interaction constant, θ_W is the Weinberg angle. In the Weinberg-Salam model $g_V = -1/2 + 2x_W$, $g_A = -1/2$, $x_W \equiv \sin^2\theta_W$.

The scattering cross section of the process is defined by:

$$\sigma = \frac{4m_e^2\varepsilon'}{f\varepsilon}\overline{\sum_{if}}\left|M_{fi}\right|^2 = \frac{\alpha^2\varepsilon'}{4f\varepsilon Q^4}R, \qquad (2)$$

where the summation $\overline{\sum_{if}} = \overline{\sum_{if(L)}}\ \overline{\sum_{if(H)}}$ imply the average over the initial spin projection states and the summation over the final spin projection states, which are performed for both lepton (L) and nucleus (H), and we call briefly the summation. After the summation we obtain the results for the expression after the second equality in (2) as follows [3, 4]:

$$R = R_F + R_{FZ} + R_Z, \qquad (3)$$

$$R_F = L_{\alpha\beta}H_F^{\alpha\beta}, \quad H_F^{\alpha\beta} \equiv \overline{\sum_{if(H)}} J_F^{\alpha*}J_F^\beta \qquad (4a)$$

$$R_{FZ} = \lambda\left(L_{\alpha\beta}^1 H_{FZ}^{\alpha\beta} + \text{conj.}\right) = 2\lambda\,\mathrm{Re}\,L_{\alpha\beta}^1 H_{FZ}^{\alpha\beta},$$

$$H_{FZ}^{\alpha\beta} \equiv \overline{\sum_{if(H)}} J_F^{\alpha*}J_Z^\beta, \qquad (4b)$$

$$R_Z = \lambda^2 L_{\alpha\beta}^2 H_Z^{\alpha\beta}, H_Z^{\alpha\beta} \equiv \overline{\sum_{if(H)}} J_Z^{\alpha*}J_Z^\beta. \quad (4c)$$

The quantities $L_{\alpha\beta}$, $L_{\alpha\beta}^1$, $L_{\alpha\beta}^2$ are called the lepton tensors, and $H_F^{\alpha\beta}$, $H_{FZ}^{\alpha\beta}$, $H_Z^{\alpha\beta}$ are nuclear tensors, which are also called the hadron tensors when considering the hadrons instead of nuclei. We see that the quantities R_F, R_{FZ}, R_Z are the contraction products of a lepton tensor with a nuclear tensor. After performing the summation over lepton states $\overline{\sum_{if(L)}}$, we obtain:

$$L_{\alpha\beta} = \mathrm{Sp}\left[\gamma_\alpha\left(1+\gamma_5\hat{S}'\right)\left(m_e+\hat{K}'\right)\gamma_\beta\left(1+\gamma_5\hat{S}\right)\left(m_e+\hat{K}\right)\right] \quad (5a)$$

$$L_{\alpha\beta}^1 = \mathrm{Sp}\left[\gamma_\alpha\left(1+\gamma_5\hat{S}'\right)\left(m_e+\hat{K}'\right)\gamma_\beta\left(g_V+g_A\gamma_5\right)\left(1+\gamma_5\hat{S}\right)\left(m_e+\hat{K}\right)\right]$$
$$= g_V L_{\alpha\beta} + g_A L'_{\alpha\beta}, \qquad (5b)$$

$$L'_{\alpha\beta} = \mathrm{Sp}\left[\gamma_\alpha\left(1+\gamma_5\hat{S}'\right)\left(m_e+\hat{K}'\right)\gamma_\beta\left(\gamma_5+\hat{S}\right)\left(m_e+\hat{K}\right)\right], \quad (5c)$$

$$L_{\alpha\beta}^2 = \mathrm{Sp}\begin{bmatrix}\gamma_\alpha\left(1+\gamma_5\hat{S}'\right)\left(m_e+\hat{K}'\right)\gamma_\beta\left(g_V+g_A\gamma_5\right) \\ \times\left(1+\gamma_5\hat{S}\right)\left(m_e+\hat{K}\right)\left(g_V-g_A\gamma_5\right)\end{bmatrix}$$

$$= \left(g_V^2+g_A^2\right)L_{\alpha\beta} + 2g_V g_A L'_{\alpha\beta}. \qquad (5d)$$

Thus there remains the summation over nuclear states $\overline{\sum_{if(H)}}$, which we shall perform in the following.

III. MULTIPOLE EXPANSION FOR THE SCATTERING CROSS SECTION

Our multipole expansion for R_F essentially concides with the results of Weigert and Rose [1], but there are some changes for being compatible with high energies [4]. At energies of order *GeV*, the contribution of R_{FZ} is of several percents from R_F, and the contribution from R_Z is of the same order compared to R_{FZ}, so R_Z can be neglected.

We employ a Cartesian coordinate system with the following unit vectors:

$\mathbf{e}_z$ = $\underline{\mathbf{q}}$, $\mathbf{e}_y$ = $\underline{\mathbf{k}\times\mathbf{k}'}$, $\mathbf{e}_x$ = $\underline{(\mathbf{k}\times\mathbf{k}')\times\mathbf{q}}$, where $\underline{\mathbf{a}} \equiv \mathbf{a}/|\mathbf{a}|$,

i.e. the *OZ* axis is along the direction of the momentum transfer **q**, the *OX* axis lies on the scattering plane, and the *OY* axis is perpendicular to this plane. The axis along **q** is called longitudinal, two other axes are transverse. The next step is to change to the cyclic coordinate system

$$\zeta_0 = \mathbf{e}_z\,,\ \ \zeta_{\pm1} = -\frac{1}{\sqrt{2}}(\pm\,\mathbf{e}_x + i\mathbf{e}_y),$$

and we shall expand the electromagnetic and weak currents of the nucleus into multipoles in this cyclic system

$$\rho(\mathbf{q}) = \sum_{Lm}\sqrt{4\pi(2L+1)}\,T^C_{Lm}(q),$$
$$\mathbf{J}(\mathbf{q}) = \sum_{Lmp}\sqrt{4\pi(2L+1)}\,T^p_{Lm}(q)\zeta^*_p\,,(p=0,\pm1), \quad (6)$$

The coefficient T^C_{Lm} is the Coulomb component of the expansion, three other coefficients T^p_{Lm} (p = 0, ±1) are composed of T^0_{Lm}, also denoted by $T^{||}_{Lm}$, which is called the longitudinal component, and two coefficients $T^{\pm1}_{Lm}$ corresponding to the transverse components, which can be written as $T^{\pm1}_{Lm}\equiv-(T^E_{Lm}\pm T^M_{Lm})/\sqrt{2}$, where T^E_{Lm} is the electric component and T^M_{Lm} is the magnetic ones. The inverse expressions of (6) are

$$T^C_{Lm}(q) = i^L\int\rho(\mathbf{r})B^C_{Lm}(q,\mathbf{r})d^3\mathbf{r}\,,$$

$$T^{||}_{Lm}(q) = i^{L-1}\int\mathbf{J}(\mathbf{r}).\mathbf{B}^{||}_{Lm}(q,\mathbf{r})d^3\mathbf{r}\,,$$

$$T^E_{Lm}(q) = i^{L+1}\int\mathbf{J}(\mathbf{r}).\mathbf{B}^E_{Lm}(q,\mathbf{r})d^3\mathbf{r}\,,$$

$$T^M_{Lm}(q) = i^L\int\mathbf{J}(\mathbf{r}).\mathbf{B}^M_{Lm}(q,\mathbf{r})d^3\mathbf{r}\,, \quad (7)$$

where B^C_{Lm}, $\mathbf{B}^{||}_{Lm}$, $\mathbf{B}^E_{Lm}$ and $\mathbf{B}^M_{Lm}$ are the basic multipole fields (of a vector field) [5], of the Coulomb, longitudinal, electric and magnetic types respectively.

We rewrite here the general form of the structure of the electromagnetic and weak currents in the unified theory [6]:

$$J^\alpha_F = V^\alpha_{00} + V^\alpha_{10}\,, \quad (8a)$$

$$J^\alpha_Z = V^\alpha + A^\alpha,\ \ V^\alpha = \beta^{(0)}_V V^\alpha_{00} + \beta^{(1)}_V V^\alpha_{10}\,,$$
$$A^\alpha = \beta^{(0)}_A A^\alpha_{00} + \beta^{(1)}_A A^\alpha_{10}\,, \quad (8b)$$

where V^α is the vector weak current, A^α is the pseudovector weak current, which is also called the axial (weak) current, two subscripts at V^α và A^α in (8*a*) and (8*b*) express the isospin (0 and 1) together with their principal projections. In the Weinberg-Salam model we have $\beta^{(0)}_V = -\,2x_W$, $\beta^{(1)}_V = 1 - 2x_W$, $\beta^{(0)}_A = 0$, $\beta^{(1)}_A = 1$. Thus the expansions (6) and (7) correspond to three currents: when $J^\alpha = J^\alpha_F$ then $T^X_{Lm} = F^X_{Lm}$, when $J^\alpha = J^\alpha_Z$ then $T^X_{Lm} = Z^X_{Lm}$ (*Z* is *V* or *A*).

Note that in quantum theory, the currents are operators, so the expressions (6) and (7) in fact are operators. For computing the scattering cross section, we need to compute first the transition amplitude of the current (6), or equivalently, of their multipole components (7), between the initial $|i\rangle \equiv |JM\rangle$ and final $|f\rangle \equiv |J'M'\rangle$ states, i.e. the expressions $\langle J'M'|\hat{J}^\alpha|JM\rangle$ or $\langle J'M'|\hat{T}^X_{Lm}|JM\rangle$. On the other hand, $\hat{F}^X_{Lm}$ is a spherical tensor, or a multipole tensor, of the order *Lm*, so according to the Wigner-Eckart theorem in quantum mechanics, the angular momentum projection quantum numbers *M*, *m*, *M'* are gathered together into a Clebsch-Gordan coefficient, and the remaining factor will not contain them:

$$\left\langle J'M'\right|\hat{T}^X_{Lm}\left|JM\right\rangle \;=\; C^{J'M'}_{JM,Lm}\left\langle J'\right\|\hat{T}^X_L\left\|J\right\rangle. \quad (9)$$

The factor $\left\langle J'\right\|\hat{T}^X_L\left\|J\right\rangle$ is called the reduced (multipole) matrix element and will be denoted by $T^X_L \equiv \left\langle J'\right\|\hat{T}^X_L\left\|J\right\rangle$, with $T = F, V, A$ and $X = C, \|, E, M$. The quantities F^X_L, V^X_L, A^X_L are also called electromagnetic, vector and axial multipole form factors respectively.

Since the currents J^α_F and V^α are conserved, so they satisfy the continuity equation

$$\frac{\partial\rho}{\partial t} + \mathrm{div}\mathbf{J} \;=\; 0\,. \quad (10)$$

From it we derive that the longitudinal component is expressed in terms of the scalar (Coulomb) component

$$F^{\|}_{Lm} \;=\; \frac{\omega}{q}F^C_{Lm},\; V^{\|}_{Lm} \;=\; \frac{\omega}{q}V^C_{Lm}\,. \quad (11)$$

The axial current is not conserved and does not satisfy the equation (10), so there is not the relation like (11) for it.

Now we compute R_{FZ}. First of all it is easy to obtain [4]:

$$L^1_{\alpha\beta} \;=\; 4\left(g_V X_{\alpha\beta} + g_A Y_{\alpha\beta}\right), \quad (12)$$

$$X_{\alpha\beta} \;=\; K_\alpha K'_\beta + K'_\alpha K_\beta + \frac{1}{2}g_{\alpha\beta}Q^2,$$

$$Y_{\alpha\beta} \;=\; i\varepsilon_{\alpha\beta\mu\nu}K^\mu K'^\nu\,. \quad (13)$$

Next, we compute the contraction product

$$X_{\alpha\beta}H^{\alpha\beta}_{FZ} \;=\; \overline{\sum_{if(H)}}\Big\{u_C\rho^*_F\rho_Z + u_\| J^{\|*}_F J^\|_Z + \frac{1}{2}u_{C\|}\left(\rho^*_F J^\|_Z + J^{\|*}_F\rho_Z\right) + 2\varepsilon^2x^2\mathbf{J}^{t*}_F.\mathbf{J}^t_Z + 2\left(\mathbf{k}.\mathbf{J}^{t*}_F\right)\left(\mathbf{k}.\mathbf{J}^t_Z\right)$$
$$-\;(\varepsilon+\varepsilon')\left[\rho^*_F(\mathbf{k}.\mathbf{J}^t_Z) + (\mathbf{k}.\mathbf{J}^{t*}_F)\rho_Z\right] - (k_\| + k'_\|)\left[J^{\|*}_F(\mathbf{k}.\mathbf{J}^t_Z) + (\mathbf{k}.\mathbf{J}^{t*}_F)\rho^\|_Z\right]\Big\}, \quad (14a)$$

$$Y_{\alpha\beta}H^{\alpha\beta}_{FZ} \;=\; \overline{\sum_{if(H)}} i\left[(\mathbf{k}\times\mathbf{k}').\left(\rho^*_F\mathbf{J}_Z - \mathbf{J}^*_F\rho_Z\right) + (\varepsilon\mathbf{k}' - \varepsilon'\mathbf{k}).\left(\mathbf{J}^*_F\times\mathbf{J}_Z\right)\right] \quad (14b)$$

Putting $K \equiv 4\pi(2J' + 1)/(2J + 1)$ and using (6) we shall obtain for 9 terms which are present in (14*a*, *b*):

$$\overline{\sum_{if(H)}}\mathrm{Re}\,\rho^*_F\rho_Z \;=\; K\sum_L F^C_L V^C_L,$$

$$\overline{\sum_{if(H)}}\mathrm{Re}\,J^{\|*}_F J^\|_Z \;=\; K\sum_L F^\|_L V^\|_L,$$

$$\overline{\sum_{if(H)}}\mathrm{Re}\left(\rho^*_F J^\|_Z + J^{\|*}_F\rho_Z\right) \;=\; K\sum_L\left(F^C_L V^\|_L + F^\|_L V^C_L\right)$$

$$\overline{\sum_{if(H)}}\mathrm{Re}\,\mathbf{J}^{t*}_F.\mathbf{J}^t_Z \;=\; K\sum_L\left(F^E_L V^E_L + F^M_L V^M_L\right),$$

$$\overline{\sum_{if(H)}}\mathrm{Re}\left(\mathbf{k}.\mathbf{J}^{t*}_F\right)\left(\mathbf{k}.\mathbf{J}^t_Z\right) \;=\; \frac{1}{2}Kk^2_t\sum_L\left(F^E_L V^E_L + F^M_L V^M_L\right)$$

$$-\overline{\sum_{if(H)}}\left(\varepsilon\mathbf{k}' - \varepsilon'\mathbf{k}\right).\mathrm{Im}\left(\mathbf{J}^*_F\times\mathbf{J}_Z\right) \;=\; Ku'_T\sum_L\left(F^E_L A^M_L + F^M_L A^E_L\right) \quad (15)$$

The remaining three terms give the expansions equal to zero.

The results of calculation are

$$R_{FZ} \;=\; 8\lambda(g_V B_1 + g_A B_2), \quad (16)$$

$$B_1 \;=\; K\sum_L\Big[u_C F^C_L V^C_L + u_\| F^\|_L V^\|_L + \frac{1}{2}u_{C\|}\left(F^C_L V^\|_L + F^\|_L V^C_L\right) + u_T\left(F^E_L V^E_L + F^M_L V^M_L\right)\Big]$$
$$= K\sum_L\left[\tilde{u}_C F^C_L V^C_L + u_T\left(F^E_L V^E_L + F^M_L V^M_L\right)\right], \quad (17a)$$

$$B_2 \;=\; Ku'_T\sum_L\left(F^E_L A^M_L + F^M_L A^E_L\right), \quad (17b)$$

where the coefficients u_C, $u_\parallel$, $u_{C\parallel}$, u_T, $\tilde{u}_C$, u'_T are called the kinematic coefficients, they appeared when computing the lepton tensors $L_{\alpha\beta}$, $L^1_{\alpha\beta}$, and have the form:

$$u_C = 2\varepsilon\varepsilon' + \frac{Q^2}{2} = 2\varepsilon\varepsilon'(1 - x^2),$$

$$u_\parallel = 2k_\parallel k'_\parallel - \frac{Q^2}{2} = 2\frac{\omega^2}{q^2}\varepsilon\varepsilon'(1-x^2),$$

$$u_{C\parallel} = -2(\varepsilon k'_\parallel + \varepsilon' k_\parallel) = -4\frac{\omega}{q}\varepsilon\varepsilon'(1-x^2),$$

$$u_T = k_t^2 - \frac{Q^2}{2} = \frac{2}{q^2}(\varepsilon^2 + \varepsilon'^2 + 2\varepsilon\varepsilon' x^2)\varepsilon\varepsilon' x^2$$

$$u'_T = \varepsilon k'_\parallel - \varepsilon' k_\parallel = -\frac{2}{q}(\varepsilon + \varepsilon')\varepsilon\varepsilon' x^2,$$

$$\tilde{u}_C = u_C + \frac{\omega}{q}u_{C\parallel} + \frac{\omega^2}{q^2}u_\parallel = \frac{32\varepsilon^3\varepsilon'^3}{q^4}x^4(1-x^4), \quad (18)$$

where the coefficient $\tilde{u}_C$ is present due to the application of (11).

In order to obtain the expression for R_F we use (4*a*), and get $R_F = 4X_{\alpha\beta}H_F^{\alpha\beta}$, after that by using the computation (15) or the computed results (17), where we must substitute $J_Z^\alpha \to J_F^\alpha$, then

$$R_F = 4A$$

$$A = K\sum_L \left\{ \begin{array}{l} u_C(F_L^C)^2 + u_\parallel(F_L^\parallel)^2 + u_{C\parallel}F_L^C V_L^\parallel \\ + u_T\left[(F_L^E)^2 + (F_L^M)^2\right] \end{array} \right\}$$

$$= K\sum_L \left\{\tilde{u}_C(F_L^C)^2 + u_T\left[(F_L^E)^2 + (F_L^M)^2\right]\right\}. \quad (19)$$

This expression coincides with the results of [1], but the ways of computing are different [7, 8].

The expressions (19), (16) and (17) computing R_F and R_{FZ} achieve the expansion of the scattering cross section into the quadric forms of the multipole form factors. Each multipole form factor is a transition amplitude of the transition current component with definite angular momentum *Lm*, where the form factor in fact depends only on *L*, not on *m*. In principle the sums in above formulas are infinite, but the selection rules derived from the symmetries limit the quantity of remaining terms, which becomes not only finite but also rather few.

IV. MULTIPOLE FORM FACTORS

The multipole form factors of the nucleus are computed on the basis of some model on the nuclear structure. By putting the obtained form factors into the scattering cross section formula, the comparison with experiments will give us a verification of the model. Up to present, there have been many such verifications, but the verifications at high energies in fact are few.

In the following we present an example on the computation of the form factors, related to the nucleus ${}^6_3Li$, and the scattering is supposed to be elastic. This nucleus has spin $J = 1$, so $L = 0, 1, 2$. The selection rules allow the existence of 8 following multipole form factors: F_0^C, F_2^C, F_1^M, V_0^C, V_2^C, V_1^M, $A_1^\parallel$, A_1^E. Since the scattering is elastic so $J' = J$, we have $K = 4\pi$. The contraction products in the cross section become

$$A/4\pi = u_C\left[\left(F_0^C\right)^2 + \left(F_2^C\right)^2\right] + u_T\left(F_1^M\right)^2,$$

$$B_1/4\pi = u_C\left(F_0^C V_0^C + F_2^C V_2^C\right) + u_T F_1^M V_1^M,$$

$$B_2/4\pi = u'_T F_1^M A_1^E. \quad (20)$$

We see that $A_1^\parallel$ does not contribute to the scattering cross section.

On the other hand, the nucleus ${}^{6}_{3}Li$ has $Z = N$, so the isospin projection $m_T = 0$, which leads to $V^{\alpha}_{10} = A^{\alpha}_{10} = 0$, because these terms are proportional to m_T. Therefore we get

$$J^{\alpha}_{F} = V^{\alpha}_{00},$$
$$V^{\alpha} = \beta^{(0)}_{V} V^{\alpha}_{00} = \beta^{(0)}_{V} J^{\alpha}_{F}, \qquad (21)$$

i.e. the weak current is proportional to the electromagnetic current, this is a common characteristic of the nuclei having $Z = N$. Furthermore in the Weinberg-Salam model we have $\beta^{(0)}_{A} = 0$, so the nuclei with $Z = N$ do not have the axial current A^{α}. This is a doubtful issue, there are evidences that $\beta^{(0)}_{A} \neq 0$ and the above mentioned data become not true. We hope to talk about this topic in a report in the future.

Willey [9] was the first to present the way of computing the multipole form factors using the many-particle theory. The author started from the expression of current density which is the sum of the currents created by each nucleon, and averaged the interactions between them. After that he computed the sum by considering the distribution of nucleons in the nucleus according to the shell model. The influence of individual nucleons on observable quantities is taken into account by introducing additional parameters related to its charge and magnetic moment as follows:

$$\varepsilon_a = \frac{1}{2}(1+\tau_{a3})\varepsilon_p + \frac{1}{2}(1-\tau_{a3})\varepsilon_n,$$
$$\gamma_a = \frac{1}{2}(1+\tau_{a3})\gamma_p + \frac{1}{2}(1-\tau_{a3})\gamma_n.$$

At low energies we have $\varepsilon_p = 1$, $\varepsilon_n = 0$, $\gamma_p = 2{,}79$, $\gamma_n = -1{,}91$. At high energies, we adjust the expression by substituting the form factors of the nucleon for above quantities: $\varepsilon_p \to G_{Ep}$, $\varepsilon_n \to G_{En}$, $\gamma_p \to G_{Mp}$, $\gamma_n \to G_{Mn}$, i.e.

$$\varepsilon_a \to \hat{X}_a \equiv \frac{1}{2}(1+\tau_{3a})G_{Ep} + \frac{1}{2}(1-\tau_{3a})G_{En},$$
$$\gamma_a \to \hat{Y}_a \equiv \frac{1}{2}(1+\tau_{3a})G_{Mp} + \frac{1}{2}(1-\tau_{3a})G_{Mn}.$$

The nucleon form factors which were fitted in experiments are [10]

$$G_{Ep} = G_D \equiv \frac{1}{\left(1-Q^2/m_V^2\right)^2},$$

$$G_{En} = -1{,}91\eta G_D/(1+5{,}6\eta), \quad \eta = -Q^2/4m_N^2,$$

$$G_{Mp} = 2{,}79 G_D, \quad G_{Mn} = -1{,}91 G_D,$$

$$m_V^2 = 0{,}71\ GeV^2, \quad m_N\text{: nucleon mass.} \qquad (22)$$

The next step is to compute the multipole form factors according to such a distribution, we obtain the following

$$F^{C}_{Lm} = e\sum_{a=1}^{A} \varepsilon_a j_L(qr_a) Y_{LM}(\underline{\mathbf{r}}_a),$$

$$F^{E}_{Lm} = e\sum_{a=1}^{A}\Big\{ \varepsilon_a \frac{\omega}{q}\left[-(L+1)j_L(qr_a) + qr_a j_{L+1}(qr_a)\right] Y_{Lm}(\underline{\mathbf{r}}_a)$$
$$+ \gamma_a \frac{q}{2M}\sqrt{L(L+1)}\, j_L(qr_a)\mathbf{Y}^{L}_{Lm}(\underline{\mathbf{r}}_a).\boldsymbol{\sigma}_a \Big\}$$

$$F^{M}_{Lm} = \frac{eq}{M\sqrt{L+1}}\sum_{a=1}^{A}\Big\{ \varepsilon_a \begin{bmatrix} \sqrt{L}\, j_{L-1}(qr_a)\mathbf{Y}^{L-1}_{Lm}(\underline{\mathbf{r}}_a) \\ +\sqrt{L+1}\, j_{L+1}(qr_a)\mathbf{Y}^{L+1}_{Lm}(\underline{\mathbf{r}}_a) \end{bmatrix}.\mathbf{l}_a$$
$$+ \frac{\sqrt{L(L+1)}}{2}\gamma_a \begin{bmatrix} \sqrt{L+1}\, j_{L-1}(qr_a)\mathbf{Y}^{L-1}_{Lm}(\underline{\mathbf{r}}_a) \\ -\sqrt{L}\, j_{L+1}(qr_a)\mathbf{Y}^{L+1}_{Lm}(\underline{\mathbf{r}}_a) \end{bmatrix}.\boldsymbol{\sigma}_a \Big\}. \qquad (23)$$

By computing the transition matrix element and using the Wigner-Eckart rule of reduction, we get

$$F^{C}_{0} = \frac{1}{\sqrt{\pi}} J_0\left[X_T + \frac{q^2}{8M^2}\left(X_T - 2Y_T\right)\right],$$

$$F_2^C = 0,$$

$$F_1^M = \frac{\sqrt{2}\,q}{\sqrt{3\pi}M} J_0 Y_T, \tag{24}$$

$$J_0 \equiv \left(1-\frac{2}{3}X\right)e^{-X}, \quad X \equiv q^2/4\beta = 26{,}58q^2,$$

$$X_T = G_{EN}^S \equiv \left(G_{Ep}+G_{En}\right)/2,$$

$$Y_T = G_{MN}^S \equiv \left(G_{Mp}+G_{Mn}\right)/2.$$

The weak form factors in this case are proportional to the electromagnetic form factors, as mentioned above, so there is no need to write them down.

V. CONCLUSION

We performed the multipole expansion for the transition currents in the nucleus into the multipole form factors and expressed the cross section in terms of them. These factors are the reduced matrix elements of the multipole components which appear when expanding the transition currents. The multipole form factors are the simplest components of the transition currents, which can be computed directly from the nuclear structure models. Therefore the method of multipole expansion allows us to obtain the more detailed information about nuclear structure, first of all the information which manifests only in the high energy processes. It also opens new perspectives in checking the unified electroweak interaction model. The computation of the multipole form factors for nuclei as well as the evaluation of the parameters of the electroweak theory will become an extensive area of the nuclear structure theory.

One of the authors (L.Z.P.) gratefully acknowledges Professor K.A. Gridnev (Saint Petersburg State University, Russia) for helpful discussions while performing some calculations.

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